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# Correction des effets de gaine pour la mesure de la densité des électrons par sonde micro-onde

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# Presentation outline

1. Introduction and context
2. The correction algorithm
  - a) The plasma-wall structure model
  - b) Emitted EM field of the probe
3. Experimental assessment
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  - b) Weakly-magnetized plasma thruster
4. Conclusion and discussion

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## **A sheath correction method for electron density measurements with the microwave resonant curling probe**

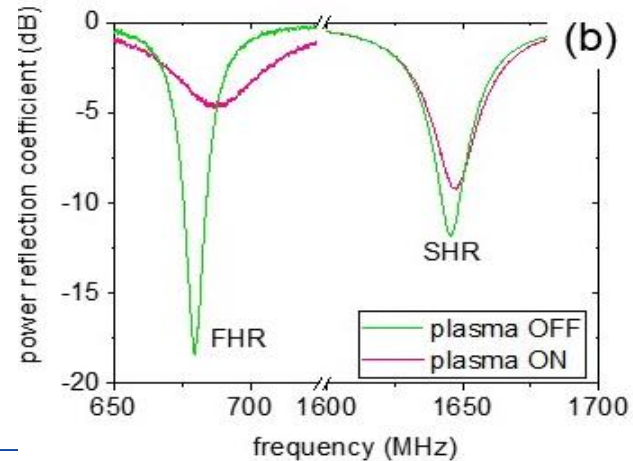
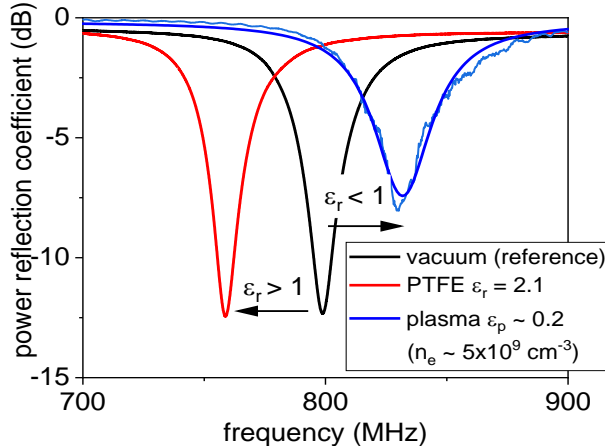
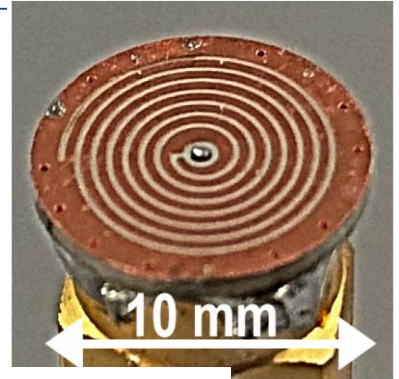
Federico Boni<sup>1,\*</sup>, Victor Désangles<sup>1</sup> and Julien Jarrige<sup>2</sup>

# The microwave resonant curling probe

- Previous studies [Liang2011, Arshadi2017, Ogawa2020]

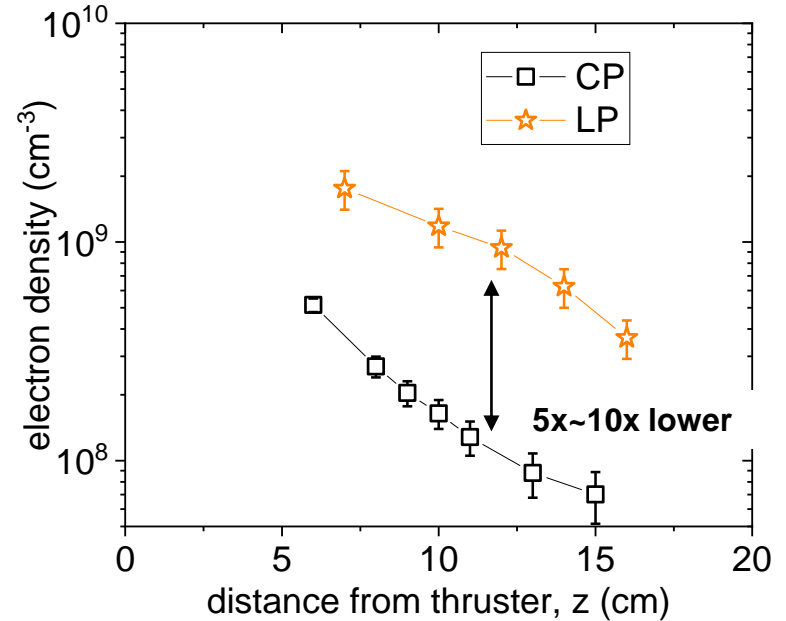
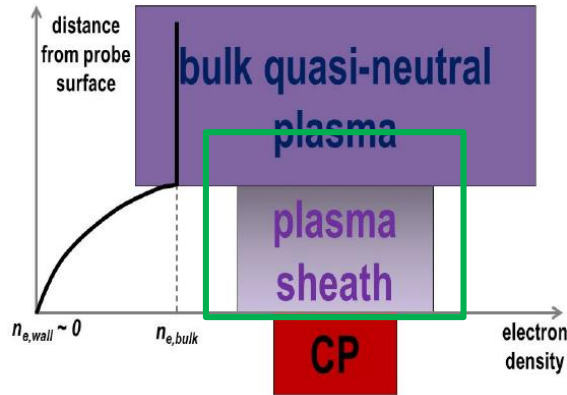
- **Principle**

- Excitation with microwaves  $\sim$  GHz /  $\sim$  mW
- Resonance frequency  $f_r$  depends on  $\epsilon_r$  of medium in contact with probe
- Absolute calibration of the probe response  $n_e \propto \Delta f$  (valid in collisionless and weakly-magnetized plasmas) [Boni2021, Boni2023]
- Use of second harmonic (SHR) to extend the measurable density range



# Problem statement

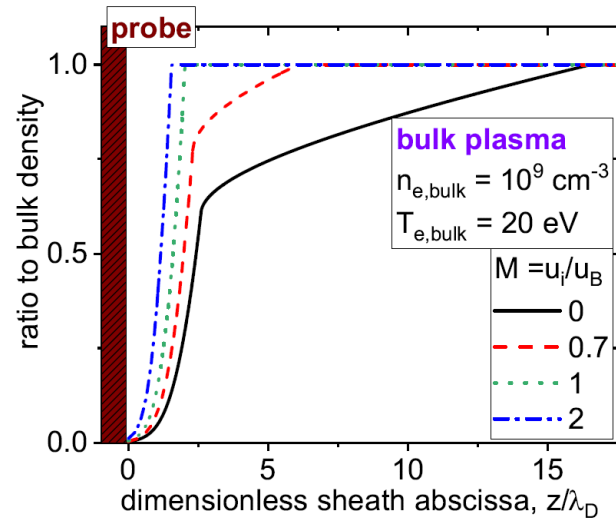
- *In-situ* measurement
- Probe electrically floating
- Formation of an (electron-depleted) sheath around the probe
- Underestimation of the measured electron density



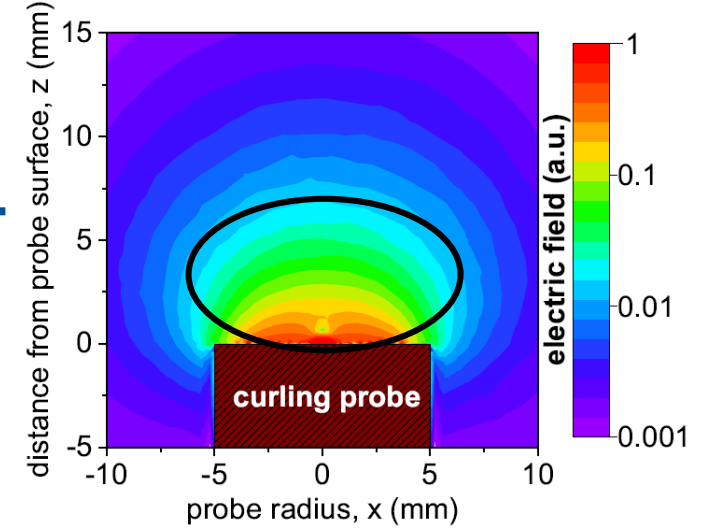
- **Sheath perturbation not negligible** when **sheaths** are **thick** compared to probed volume from CP
- **Underestimation** of raw measured density up to **90%** (worst case)

# How to mitigate plasma sheath effects?

Electron density profile due to plasma-wall interaction



The characteristic decay length ( $\sim \text{mm}$ ) of the EM field emitted by the CP



Development of a plasma sheath correction method

# The plasma-wall structure model

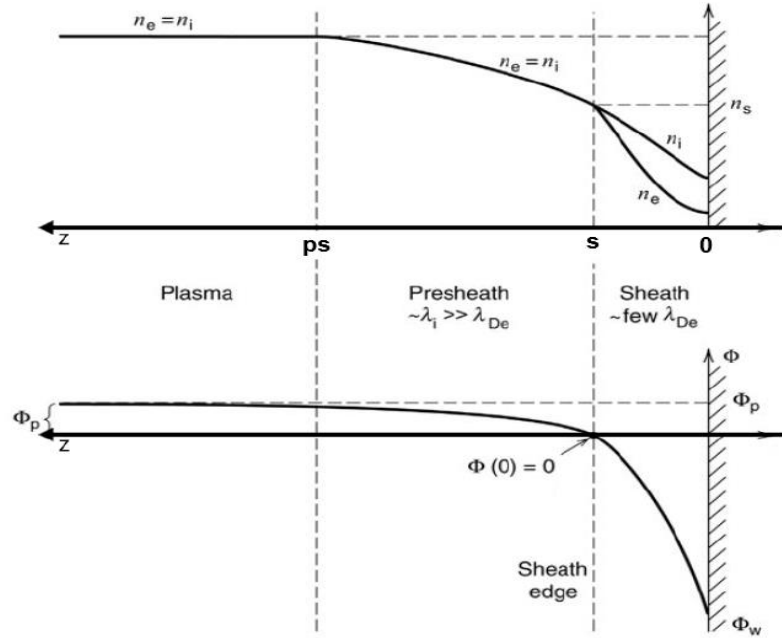
## Main characteristics of the plasma-probe environment

- Electron densities  $10^8$ - $10^{11}$   $\text{cm}^{-3}$
- Electron temperatures 5-30 eV
- Possibility to account for subsonic or supersonic ions
- CP electrically floating

$$\lambda_D = 50 \mu\text{m} - 4 \text{ mm}$$

## Hypothesis of the model

- 1D
- Collisionless and weakly-magnetized
- Maxwellian electrons (bulk+sheath =  $T_{e,\text{bulk}}$ )
- Singly charged ions with  $T_i=0$
- Ion velocity drives sheath formation
- SEE neglected
- Bohm criterion respected [Bohm1949]
  - $M_i = u_i/c_s < 1$ : bulk plasma – presheath + sheath – wall
  - $M_i = u_i/c_s > 1$ : bulk plasma – sheath – wall



[Lieberman2005]

$$c_s = \sqrt{\frac{eT_e}{M_i}}$$

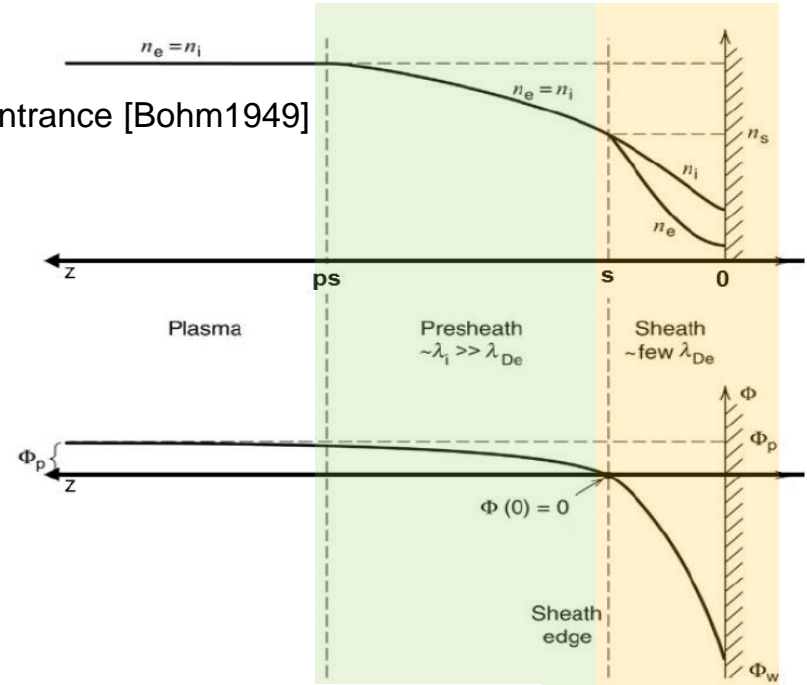
# The presheath region

When  $M_i = u_i/c_s < 1$  in the bulk plasma  $\rightarrow$  presheath

Acceleration region in which ions reach sound speed at the sheath entrance [Bohm1949]

- Quasi-neutral region, modeled according to [Riemann1991] scales with ion-neutral mean free path  $\rightarrow \lambda_i \propto 1/p$
- Potential drop  $\rightarrow \frac{\Delta\phi_{ps}}{T_e} = -\frac{1}{2}(1 - M^2)$
- Density drop  $\rightarrow \alpha_{ps} = \frac{n_s}{n_b} = e^{\left[-\frac{1}{2}(1 - M^2)\right]}$
- If  $M_i=0 \rightarrow \Delta\phi_{ps} = T_e/2$  and  $\alpha_{ps} = 0,61$  [Lieberman2005]

Presheath effect accounted for through presheath/sheath density drop  $\alpha_{ps}$



Presheath [Riemann1991] Sheath [Lieberman2005]

# The sheath region

- Electrostatic potential equation (Poisson + Maxwellian electrons + conservation of ion flux [Lieberman2005])
- E field at sheath edge is non-zero [Godyak1990]
- $d\phi/dz$  numerically integrated to obtain  $\phi(z)$
- **Solved for  $n_b = 10^8 - 10^{11} \text{ cm}^{-3}$ ,  $T_e = 1 - 50 \text{ eV}$ ,  $E_{ki} = 5 - 300 \text{ eV} \rightarrow 0 \leq M_i \leq 10$**

$$\frac{d\Phi}{dz}(z) = -\sqrt{\frac{2en_s}{\epsilon_0} \left\{ k_B T_e \left( \exp\left(\frac{e\Phi(z)}{k_B T_e}\right) - 1 \right) + 2E_{ki} \left( \sqrt{1 - \frac{e\Phi(z)}{E_{ki}}} - 1 \right) \right\}} \frac{k_B T_e}{e\lambda_D}$$

Sheath thickness is defined as the length over which a potential drop of modified  $\phi_{wall}$  occurs

$$\phi_{wall} = k_B T_e \ln \frac{4u_s}{u_{el}}$$

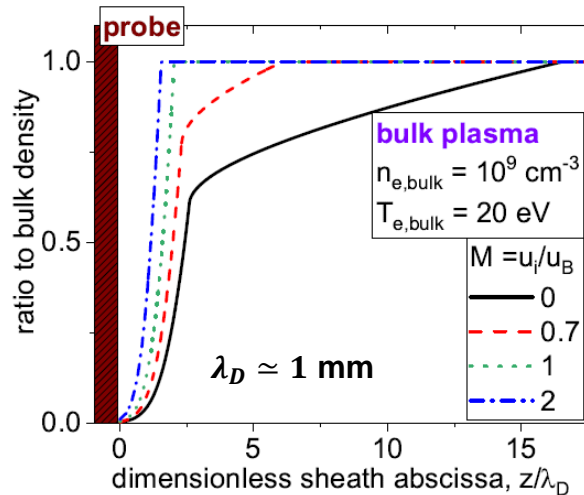
Ion velocity at sheath edge  $M_i \geq 0$



# The dimensionless sheath thickness

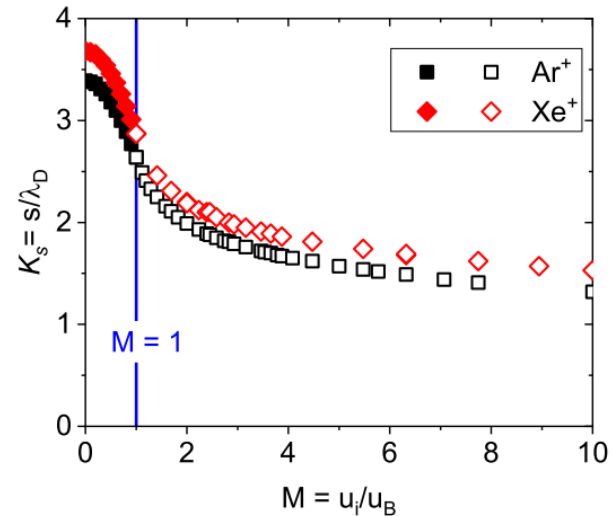
## Dimensional sheath thickness ( $s$ )

- dependent upon bulk conditions ( $n_b, T_e, M_i$ ) and gas
- For  $M_i = 0$ , thickness =  $16\lambda_D$
- For  $M_i = 2$ , thickness =  $1,6\lambda_D$



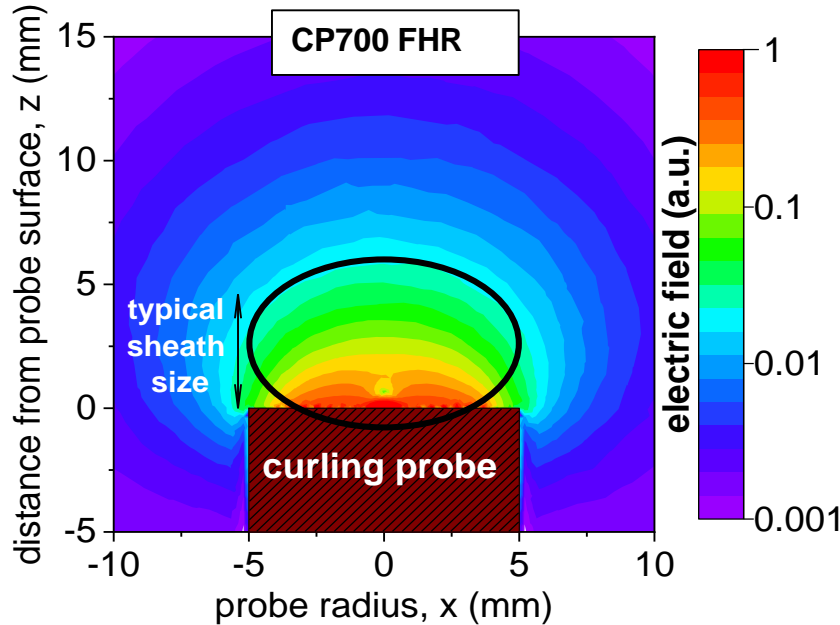
## Dimensionless sheath thickness $K_s = \frac{s}{\lambda_D}$

- Only dependent upon  $M_i$  and gas
- Simpler use in the algorithm
- Does not include presheath width



# Emitted EM field of the curling probe

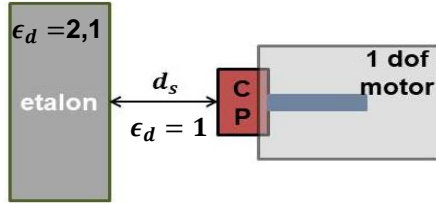
- 3D numerical simulations (COMSOL): frequency sweep studies
- EM field emitted by the CP extends for several mm above the probe surface
- A change in permittivity in the probed volume affects the resonance frequency



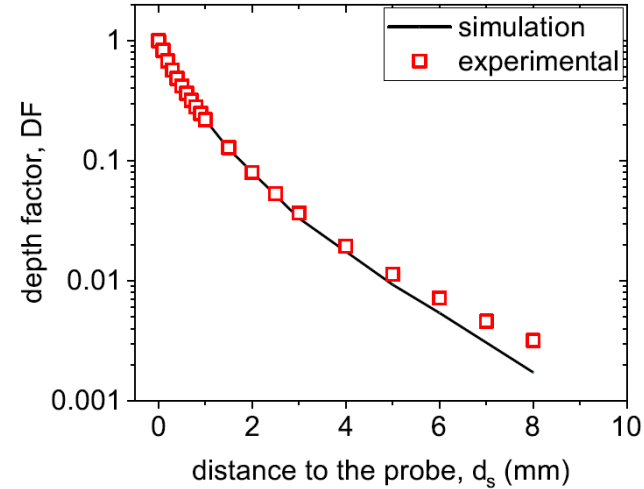
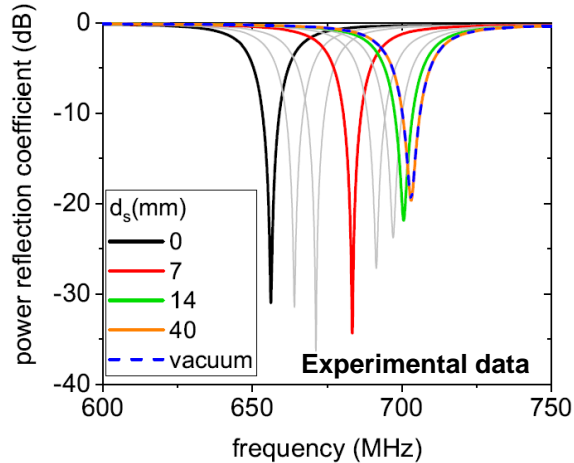
- E field decreases by 10x times at  $z \sim 5$  mm
- Spatial distribution is harmonic/probe dependent

# Definition of the dielectric depth factor of a CP

- This case can be reproduced numerically and experimentally
- Thickness of etalon >> characteristic decay length of emitted EM field



$$DF(d_s) = \frac{\Delta f(d_s)}{\Delta f(d_s = 0)} = \frac{f_r(d_s) - f_0}{f_r(d_s = 0) - f_0},$$



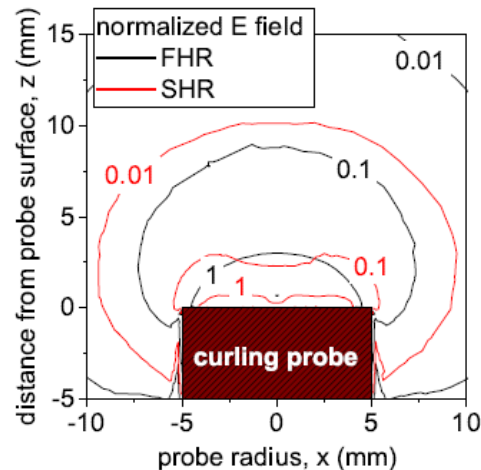
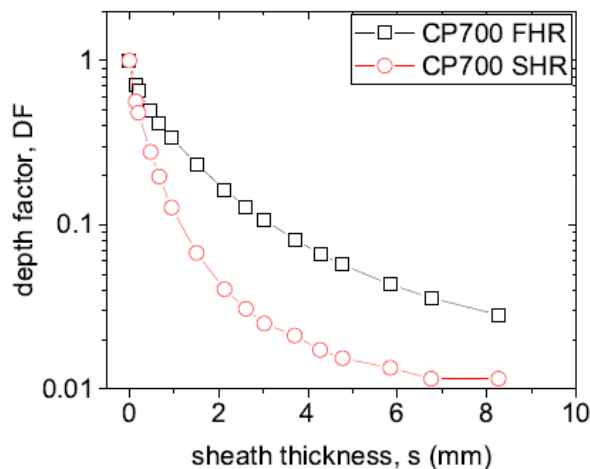
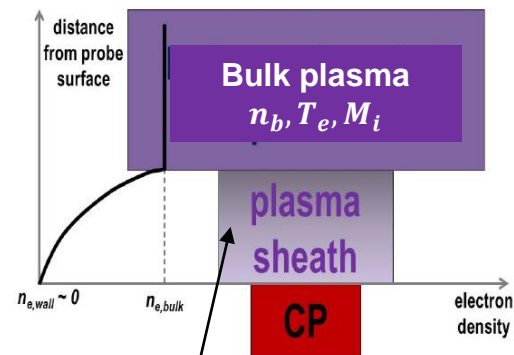
- Direct relation btw  $\Delta f$  (freq shift) and  $d_s$  (dist etalon-probe)
- Excellent agreement simulations / experiments
- **3D numerical simulations are considered representative of CP behavior in the presence of non-homogenous permittivity profile**

# Definition of the plasma depth factor of a CP

$n_b = 10^8 - 10^{11} \text{ cm}^{-3}$ ,  $T_e = 1 - 50 \text{ eV}$ ,  $E_{ki} = 5 - 300 \text{ eV}$   
 $\rightarrow 20 \mu\text{m} \leq \lambda_D \leq 5 \text{ mm}$  and  $100 \mu\text{m} \leq s \leq 8 \text{ mm}$

$$DF(s) = \frac{\Delta f(s)}{\Delta f(s=0)} = \frac{n_e(s)}{n_e(s=0)} = \frac{n_{e,raw}}{n_{e,bulk}}$$

## Simulation setup



Sheath with  $n_e(z)$  obtained with model presented before (no presheath)

- DF is the probe sensitivity to the plasma sheath (equivalency btw  $\Delta f$  and  $n_e^{MEAS}$ )  
 DF uniquely depends upon sheath thickness and is harmonic/probe dependent

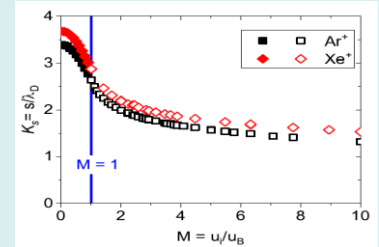
# The correction algorithm

Inputs

$n_{meas}^{RAW}$ ,  $T_e$  (estimation),  $M_i$  (estimation)

Computation

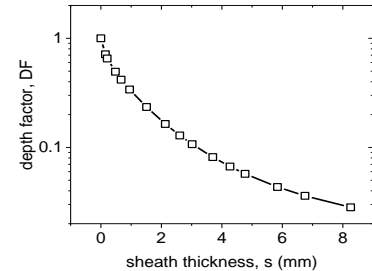
$$K_s = \text{function}(M_i, \text{gas})$$



Iterations

$$n_{bulk}^{i+1} = DF(s(n_{bulk}^i, T_e, M_i)) \cdot n_{meas}^{RAW}$$

$$s(n_{bulk}^i, T_e, M_i) = K_s(M_i) \cdot \lambda_D(n_{bulk}^i, T_e)$$



Final step

If  $M_i < 1 \rightarrow$  presheath  $\rightarrow \alpha_{ps}$  correction

Output

$n_{bulk}$

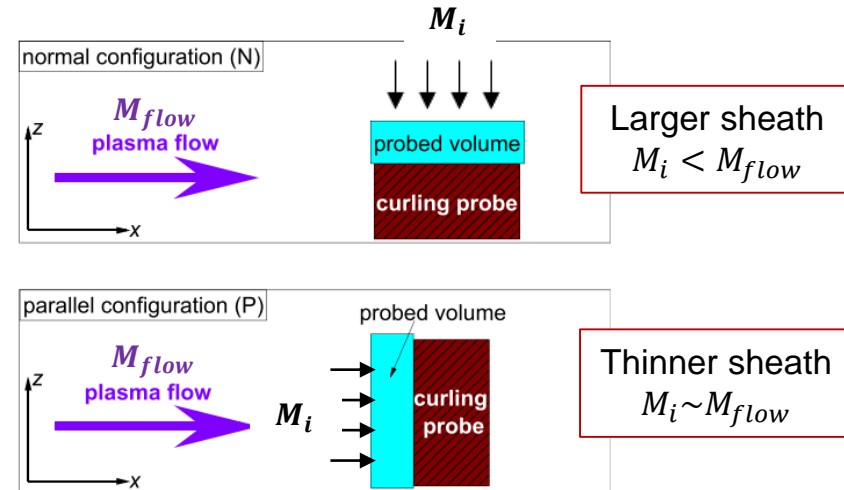
# Experimental assessment of the method

## Two plasma sources

- **ICP source (4 MHz)**
  - No B field
  - Maxwellian electrons [Esteves2022]
  - Subsonic-to-sonic ions
  
- **ECR thruster plume**
  - weakly-magnetized ( $B = 500\text{-}10\text{ G}$ )  
→ minor role of B [Chodura1982]
  - Possible deviations from Maxwellian [Correyero2017]
  - Supersonic ions

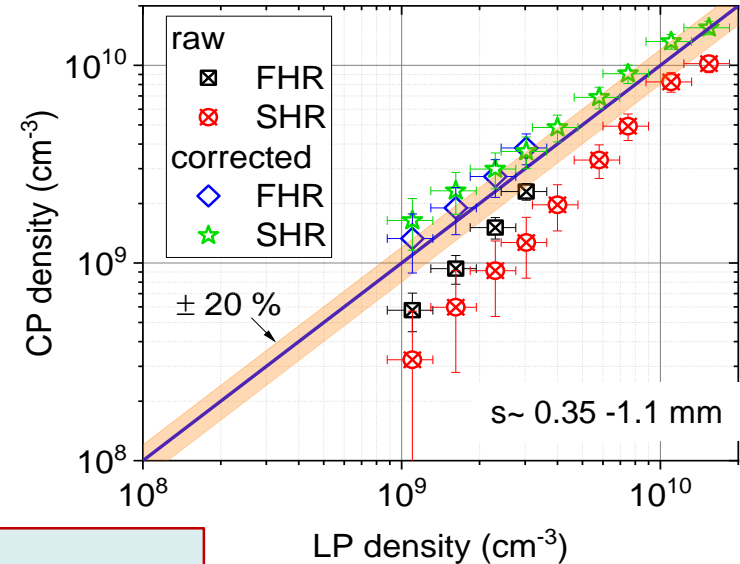
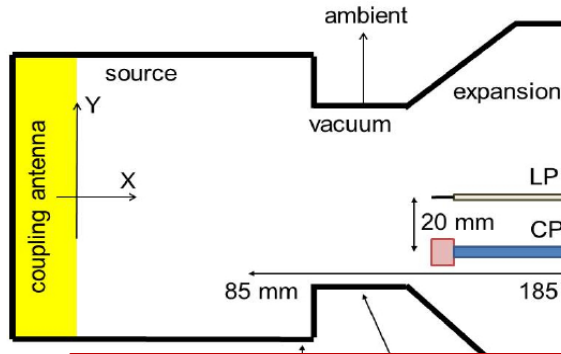
## Three comparisons

1. Curling probe vs Langmuir probe
2. FHR vs SHR of CP700  
→ same sheath thickness, different DF
3. Change of CP orientation wrt ion beam  
→ same DF, different sheath thickness



# ICP source: FHR and SHR vs LP

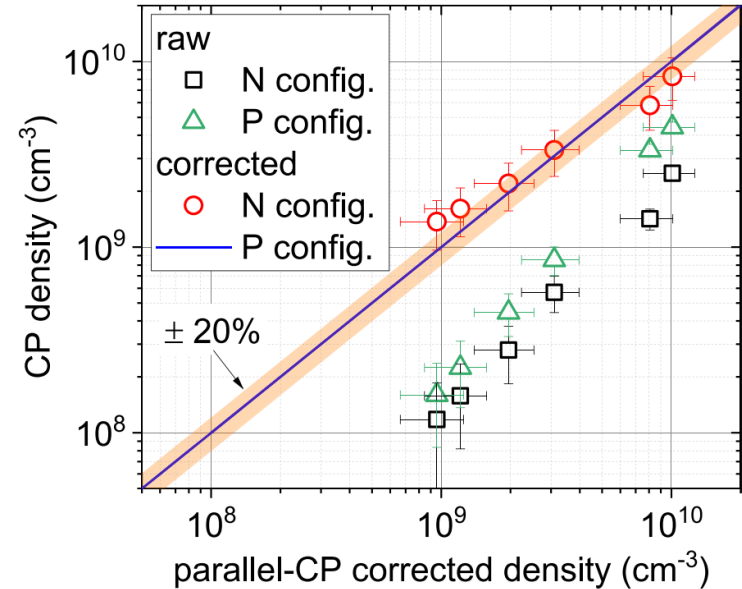
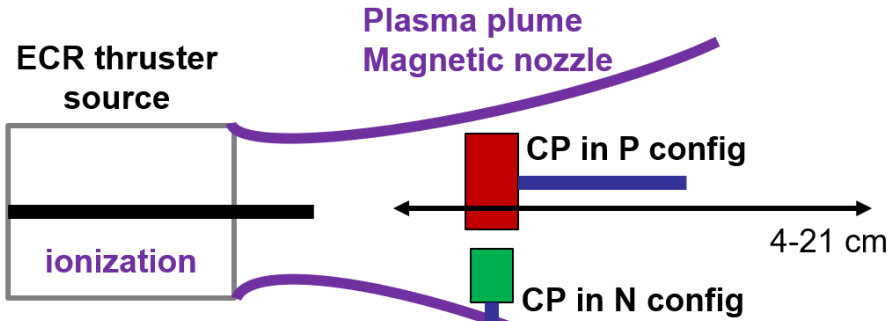
- Pressure between  $10^{-2}$ - $10^{-3}$  mbar, power between 50-100 W, argon gas
- Measurements taken far downstream the antenna (low coll. and ioniz.)
- LP post-processed using [Druyvesteyn1940]
- CP used in P configuration
- $M_i = 0,7-1,3$  &  $T_e \sim 2-4$  eV  $\rightarrow M_{corr} = 1, T_{corr} = 3$  eV



- $n_{raw}^{SHR} < n_{raw}^{FHR} \rightarrow$  consistent with their DFs
  - Corrected CP data in agreement with LP (20% discrepancy)
  - Discrepancy btw FHR and SHR reduced from 40% to 10%
- Validation of depth factor approach**  
(same plasma sheath and bulk, different harmonic)

# ECR thruster plume: N and P configurations

- Pressure  $\sim 10^{-6}$  mbar, power  $\sim 30$  W, 1 SCCM xenon
- $M_i = 1 - 3$ ,  $T_e = 20$  eV
- CP700 FHR
  - P configuration  $\rightarrow M_{corr} = M_i = 2, T_{corr} = 20$  eV
  - N configuration  $\rightarrow M_{corr} = 0.3, T_{corr} = 20$  eV



- $n_{Nconfig}^{RAW} < n_{Pconfig}^{RAW}$  due to larger sheath (discrepancy 38%)
  - $s \sim 0,7-2$  mm in P config vs  $s \sim 1-4$  mm in the N config
  - Corrected parallel and normal CP data in good agreement (discrepancy  $< 15\%$ )
- Validation of the effect of  $M_i$  on the sheath formation**  
**(same harmonic=same DF, same bulk plasma, different sheath)**



# General remarks and conclusion

- $K_s$  depends only on  $M_i$  and gas
- **DF property is directly correlated to  $n_e$  decrease due to presence of sheath**
- Semi-analytical 1D and DC model used is appropriate in the cases studied here
  - Density  $10^8 - 10^{11} \text{ cm}^{-3}$
  - Temperature 5-30 eV
  - Ion beams up to 200 eV ( $\sim 17 \text{ km/s}$  for xenon)
  - Weakly-magnetized ( $< 400 \text{ G}$ ,  $f_{ce} \sim 1 \text{ GHz}$  with CP at 1700 MHz)
  - Collisionless or weakly-collisional ( $f_c$  up to  $\sim 100 \text{ MHz}$  with CP at 700 MHz)
- Experimental observations
  - $n_{CP}^{RAW} < n_{LP}$  due to sheath effects forming around the CP
  - $n_{CP}^{CORR}$  in agreement with  $n_{LP}$
  - $n_{Nconfig}^{CORR}$  in agreement with  $n_{Pconfig}^{CORR} \rightarrow$  **ion velocity drives sheath thickness**
  - $n_{FHR}^{CORR}$  in agreement with  $n_{SHR}^{CORR} \rightarrow$  **DF property related to density decrease in sheath**

✓ Method globally validated

# Limitations of the presented model

- In the **sonic-to-supersonic regime** ( $M \gtrsim 1$ ), method **not too sensitive to  $M_{corr}$ ,  $T_{corr}$**  (discrepancies ~20-30%)
- In the **subsonic regime** ( $M < 1$ ), method is much **more sensitive to input parameters** (discrepancies ~40-60%) (due to presence of presheath → additional correction due to  $\alpha_{ps}$  that is absent in the supersonic case)
- Uncertainties on  $T_{corr}$  affect more the output accuracy compared to  $E_{ki}$  → see example below

$n_b = 10^{10} \text{ cm}^{-3}, T_e = 20 \text{ eV}, M_i = 0,7 \text{ (E}_{ki}=5 \text{ eV)}, n_{raw} = 3,3 \times 10^9 \text{ cm}^{-3}$

Input						
$T_e$ (eV)	$E_{ki}$ (eV)	$M$	$\alpha_{ps}$	$s$ (mm)	DF (s)	Estimated bulk density ( $\times 10^{10} \text{ cm}^{-3}$ )
20	5	0.7	0.77	1.08	0.43	0.99
40	5	0.5	0.69	1.63	0.32	1.48
20	2.5	0.5	0.69	1.15	0.41	1.17

- $T_e$  and  $E_{ki}$  affects  $M_i$   
→ same effect on  $\alpha_{ps}$
- $T_e$  affects sheath thickness more than  $E_{ki}$   
→ it changes the scale length ( $\lambda_D$ )

- Magnetized models should be used if  $f_p < f$  and  $f_{ce} \rightarrow f$  (B field effects cannot be neglected on  $\epsilon_p$ )
- P configuration → B field and ion velocity are always aligned → sheath is governed by velocity magnitude
- N configuration → according to [Ahedo1997], B effects should be weak  
Considering good accordance btw N and P corrected data → B effects in sheath formation in N config can be neglected here

# What can be improved

- **Inclusion of collisions and ionization** in presheath/sheath models
- **More representative IEDF** (not a Dirac as here)  
→ may help reduce uncertainties in the sonic transition region (where high uncertainties are now obtained)
- Computation of **DF with presheath** (now computed with sheath  $n_e$  drop)  
Acceptable in cases where density gradient in the sheath  $\gg$  than that presheath AND sheath  $\sim 10x$  times thinner than presheath → this could enable **more precise** determination of **CP sensitivity** to plasma sheath in **subsonic** case and **thin sheaths**
- Effect of **strong B field** at grazing incidence on **sheath formation** → magnetic presheath?
- **Non-Maxwellian electrons / non-uniform  $T_e$**  across the plasma-wall structure  
→ Maxwellian + uniform  $T_e$  could currently overestimate sheath potential drop [Kushner1985]
- Increase representativity of sheath potential drop by considering ion beam velocity and EEDF [Schroder2015]
- $\lambda_D > s_{80}$  in our case → underestimations are always non-negligible  
→ application of this method  $\uparrow$  accuracy of results
- If  $\lambda_D \leq s_{80}$  → underestimations  $< 20\%$  and the application of method could potentially  $\downarrow$  accuracy of  $n^{CORR}$

$s_{80}$  is thickness value for which DF=80%  
→ if  $s = s_{80}$  then sheath effects could be neglected, since underestimations  $< 20\%$

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**Thank you for  
your attention !**