A sheath model for arbitrary radiofrequency waveforms

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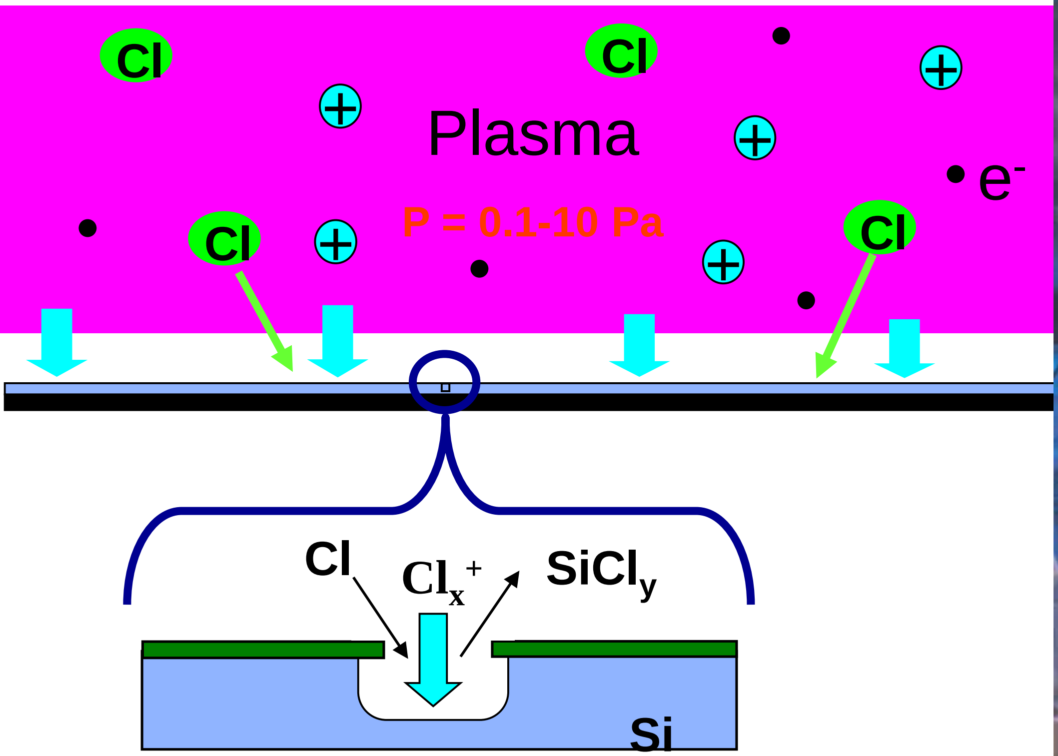
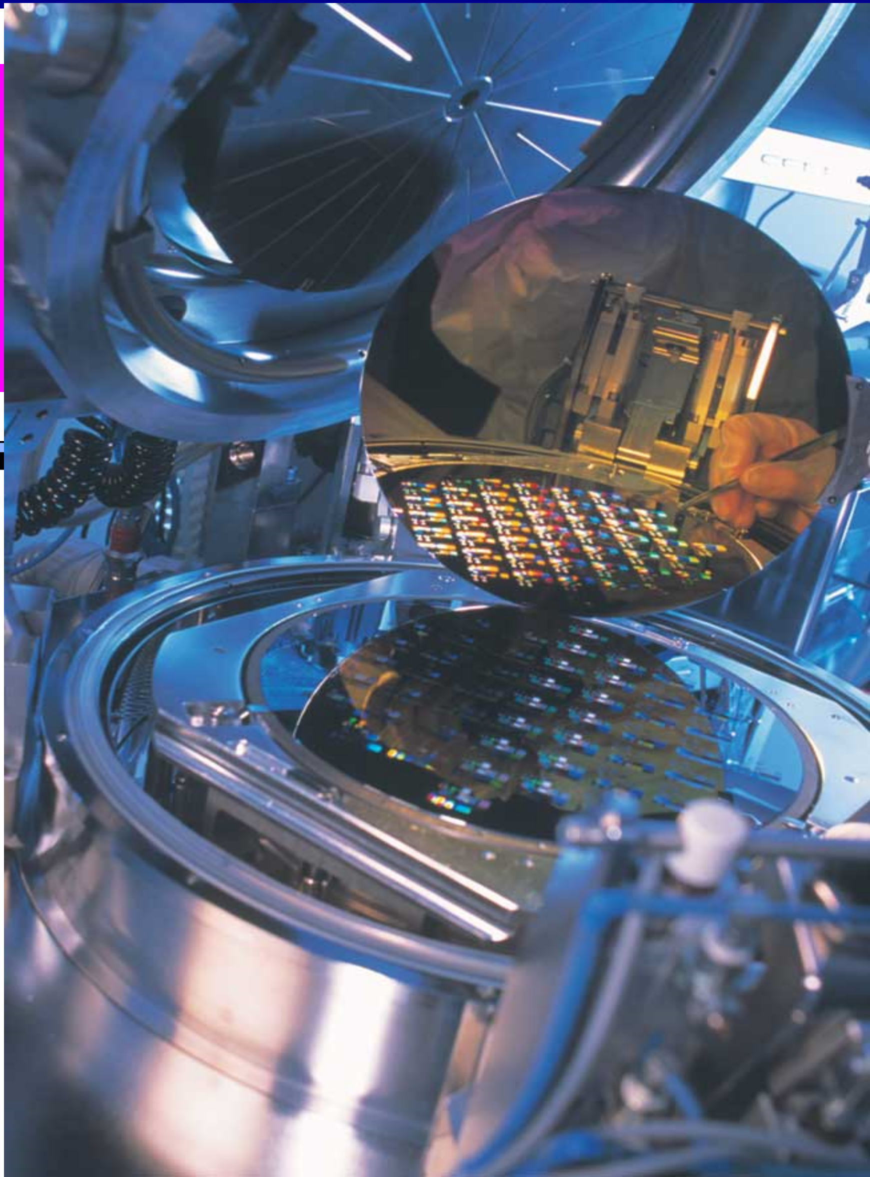
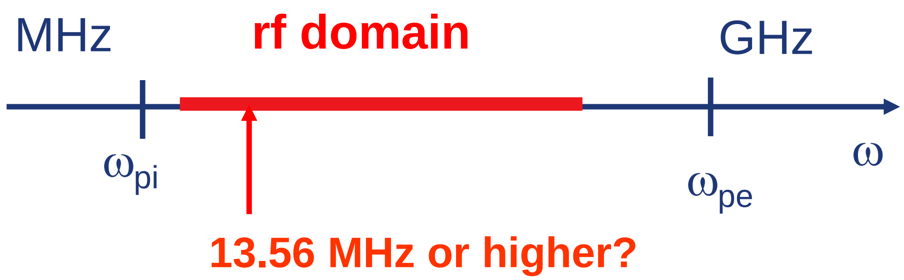
RF Plasmas in industry



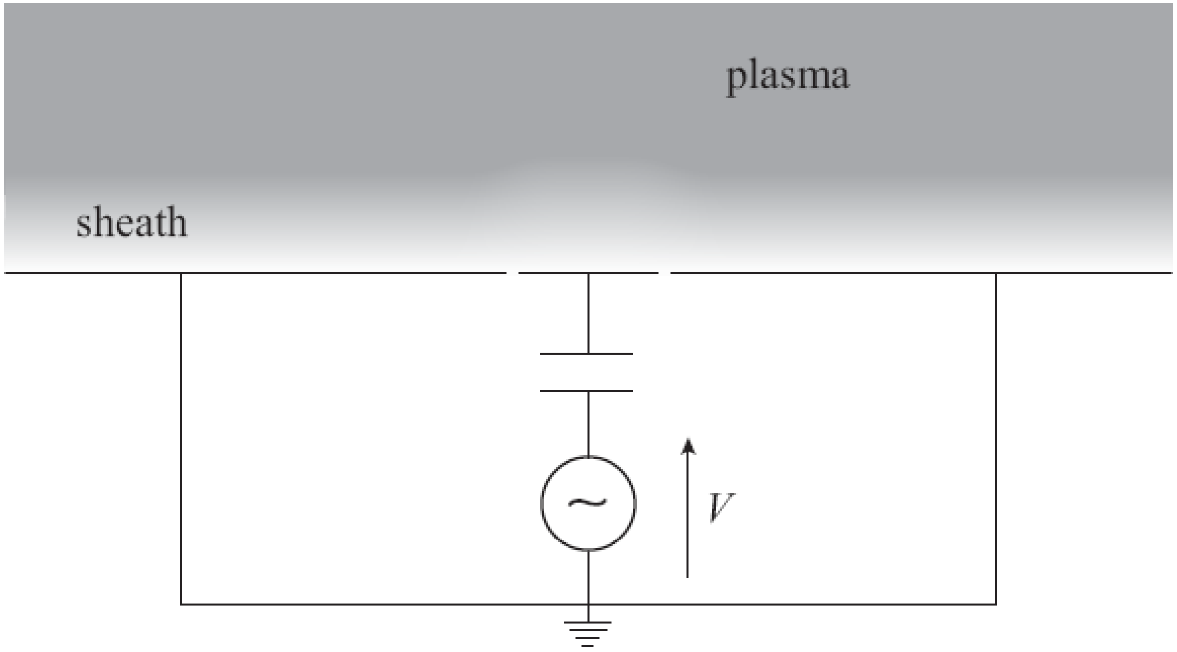
- Plasma etching
- Plasma deposition
- Plasma thrusters



Frequency domain



- **Electrons follow the rf field**
- **Ions may not follow the rf field but may respond to time-averaged field**

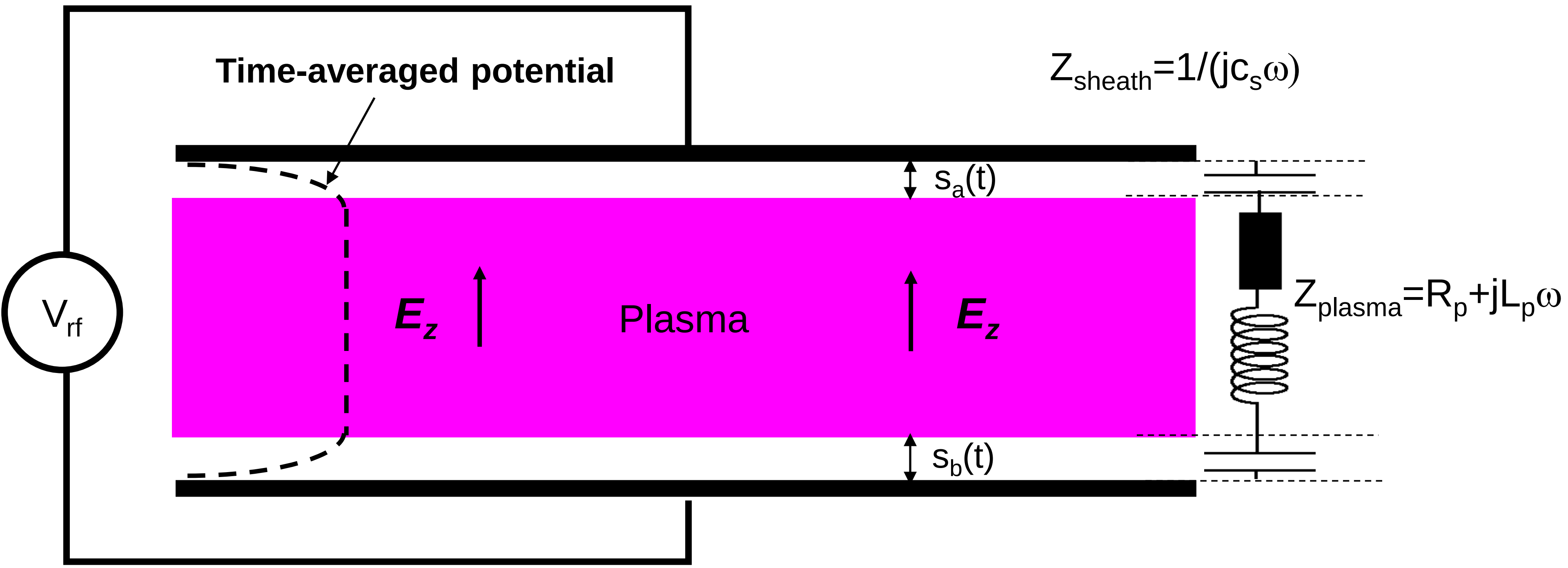


$$\omega_{pe} = \sqrt{\frac{ne^2}{m\varepsilon_0}}$$

$$\omega_{pi} = \sqrt{\frac{ne^2}{M\varepsilon_0}}.$$

Figure 4.1 A capacitively coupled electrode adjacent to a plasma excited by some unspecified external means; $V = V_1 \sin \omega t$.

Traditional capacitive discharges



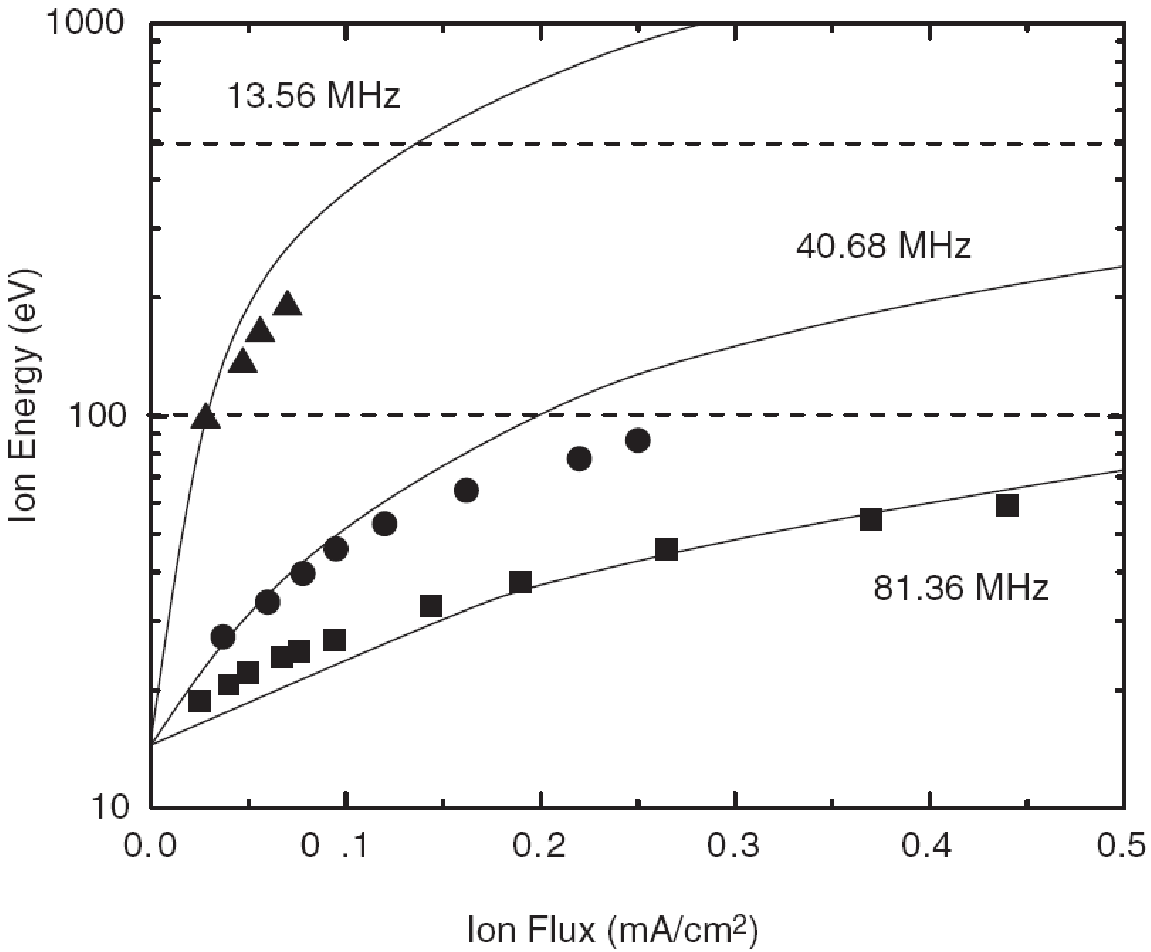
Impedance depends on :

- Voltage, V_{rf}
- Electron density, n_e
- Sheath size, s_m

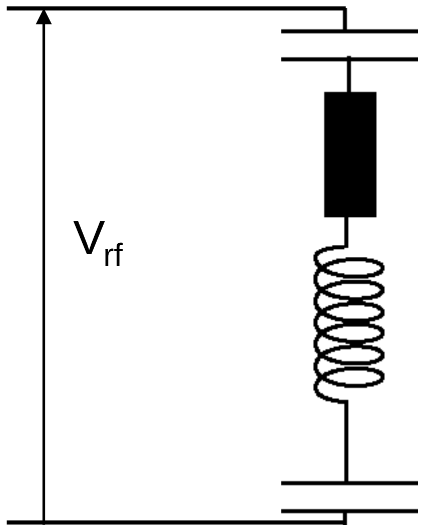
To find a self-consistent solution:

- Child law
- Particle balance
- Power balance

Dual Frequency with well separated frequencies



■●▲ : Experiments
----- : model

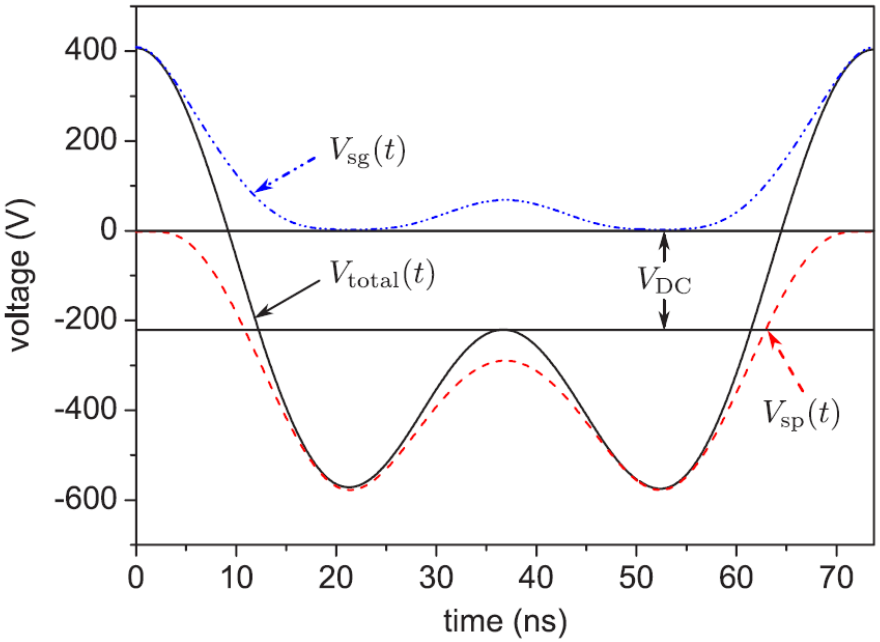


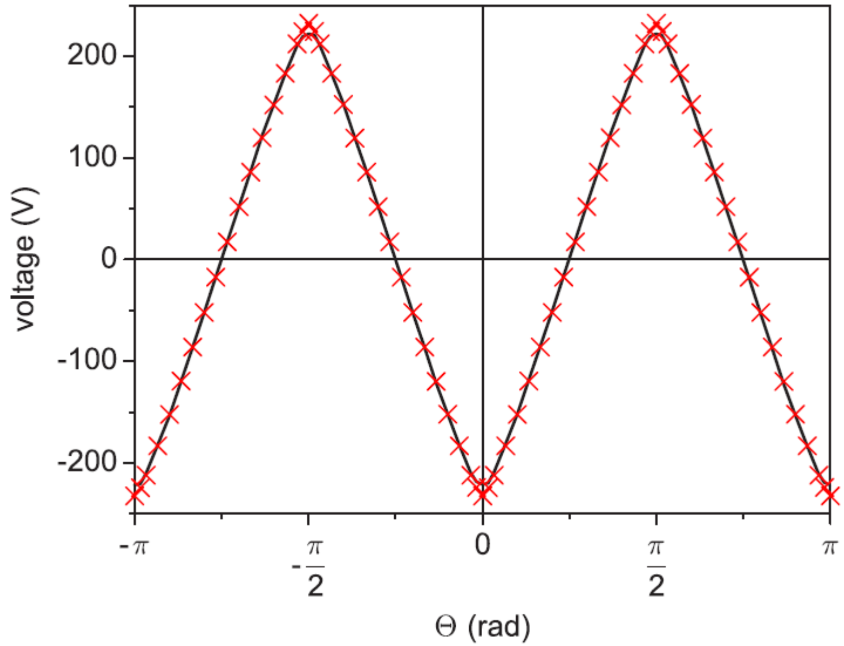
➔HF drive for high flux
➔LF bias for tunable ion energy

A. Perret et al., *Appl. Phys. Lett* ***86*** *(2005) 021501*

Dual Frequency Capacitive (DFC)

Electrical assymmetry effect

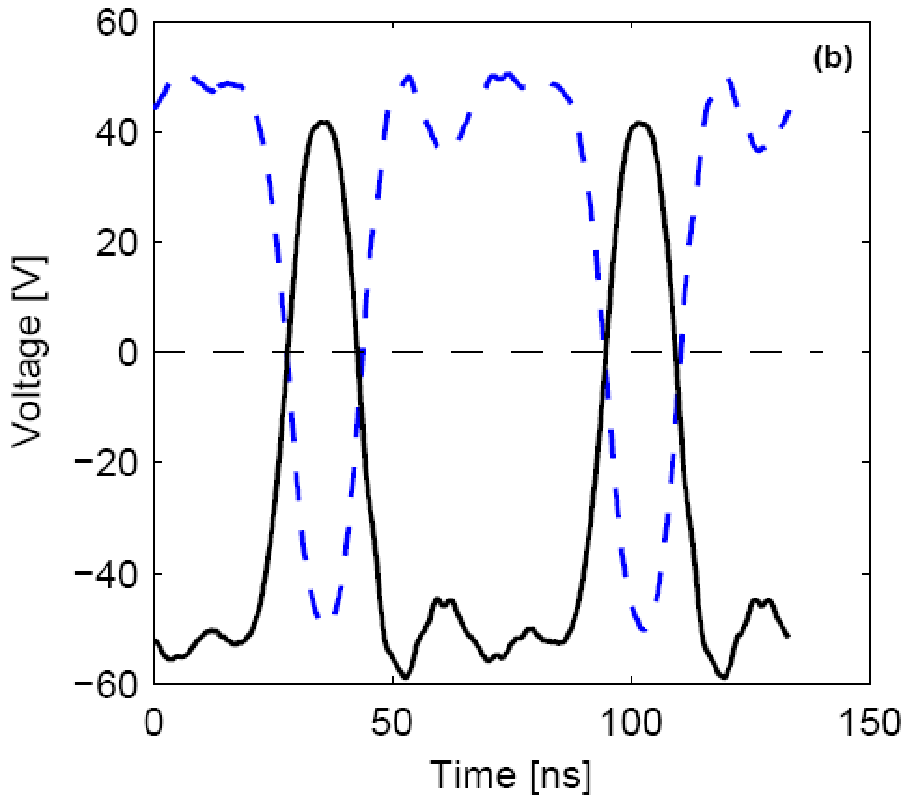


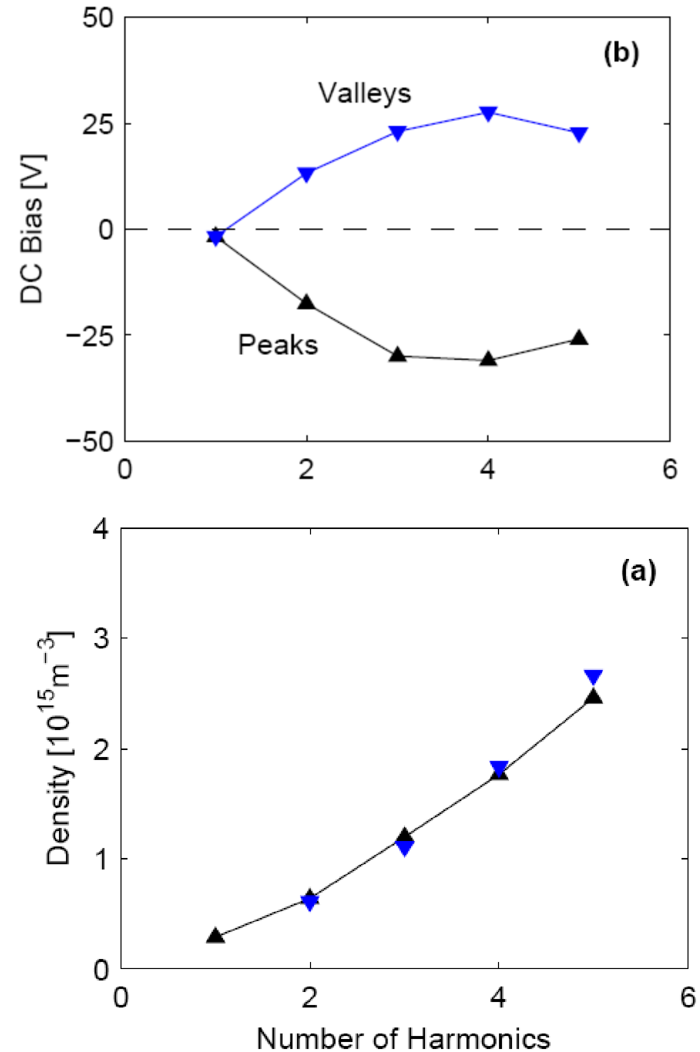


- It is possible to change the self-bias by changing the phase between the two frequencies, even when geometrically symmetrical
- The asymmetry is generated by the voltage waveform

B. G Heil, U. Czarnetzki, RP Brinkmann and T.Mussenbrock, J. Phys. D: Appl. Phys. 41 (2008) 165202

Complex waveforms





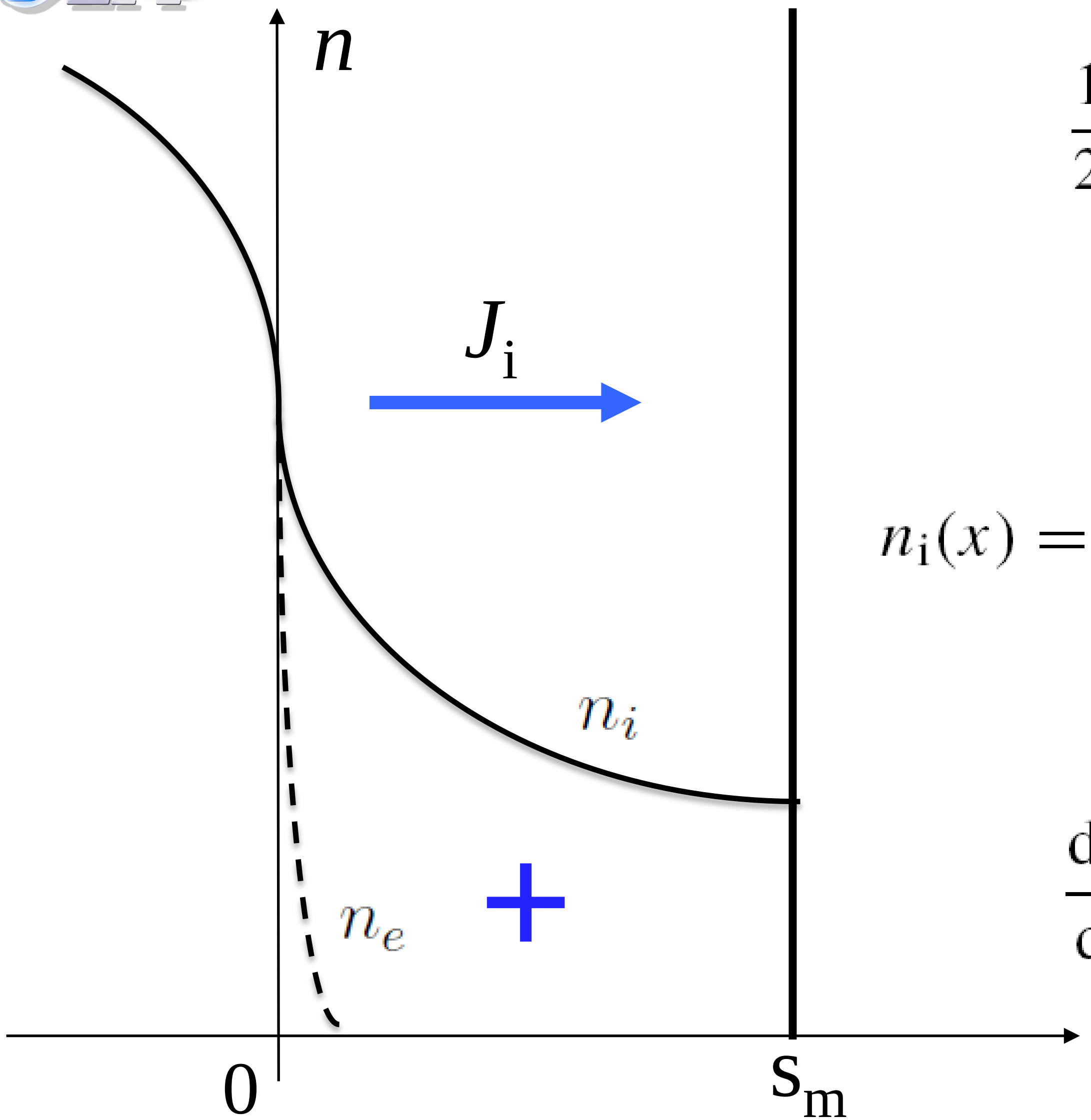
- This is a generalization to complex waveforms
- Electron heating is also affected

T. Lafleur, P.A. Delattre, E.V. Johnson, and J.P. Booth, Appl. Phys. Lett. (2012)

Motivation

- The sheath is the most important element in capacitive discharges
- Electron heating, EEDF, IEDF on the substrate are all determined by the sheath physics
- Lieberman (after Godyak) supplied an analytical model of the RF sheath for sinusoidal waveform [IEEE Trans. Plasma Sci. 16, 638 (1988)]
- Lieberman's model cannot be generalized to arbitrary RF waveforms
- In this talk we present an analytical model for arbitrary RF waveforms

The DC sheath



$$\frac{1}{2}Mu(x)^2 + e\phi(x) = 0$$

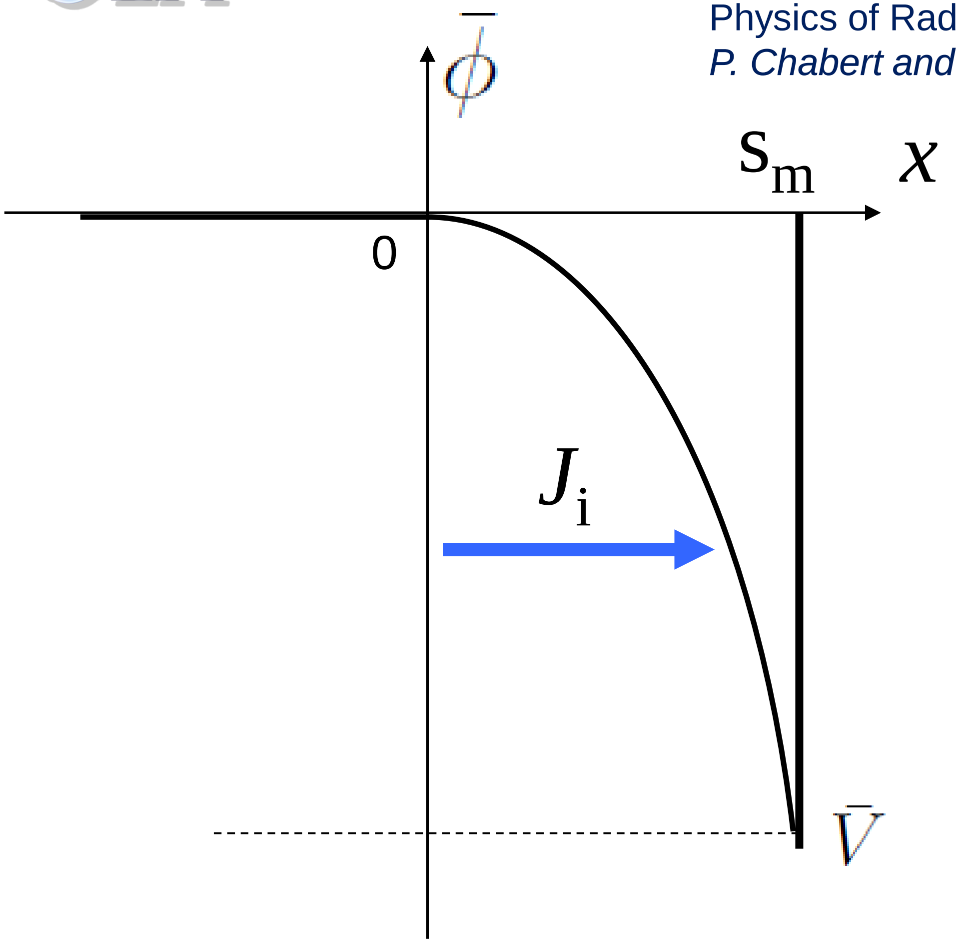
$$J_\mathrm{i} = en_\mathrm{i}(x)u(x)$$

$$n_\mathrm{i}(x) = \frac{J_\mathrm{i}}{e}\left(-\frac{2e\phi(x)}{M}\right)^{-1/2}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = -\frac{J_\mathrm{i}}{\varepsilon_0}\left(-\frac{2e\phi(x)}{M}\right)^{-1/2}$$

The DC sheath

Physics of Radiofrequency Plasmas
P. Chabert and N. Braithwaite, Cambridge University Press, 2011



$$\xi = 1 \qquad K_i = 4/(9\xi)$$

$$J_i = K_i \frac{\epsilon_0}{s_m^2} \left(\frac{2e}{M}\right)^{\frac{1}{2}} \left(-\bar{V}\right)^{\frac{3}{2}}$$

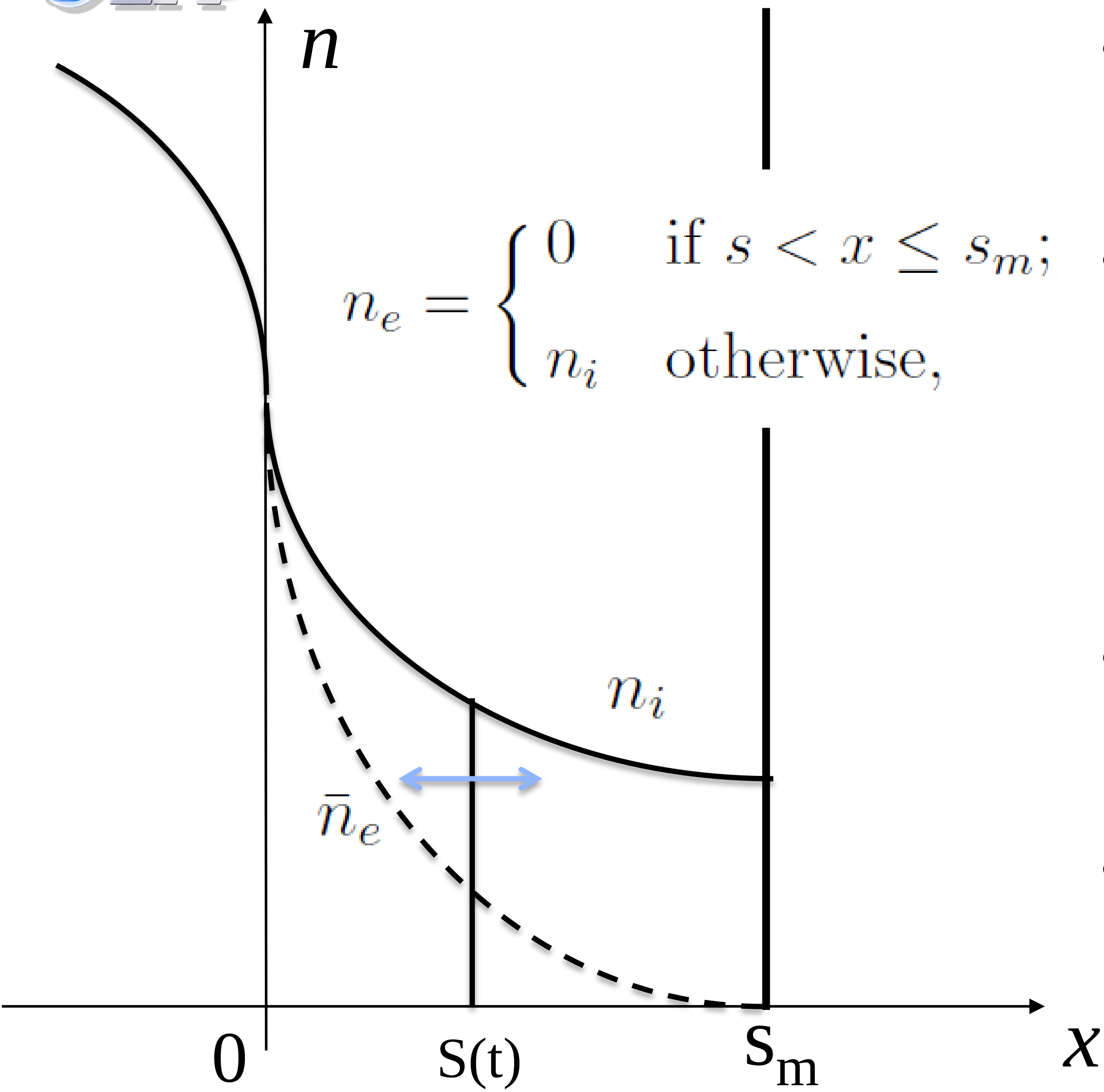
$$n_i(x) = -\frac{4}{9}\frac{\epsilon_0 \bar{V}}{\xi e s_m^2}\left(\frac{s_m}{x}\right)^{\frac{2}{3}}$$

$$\bar{\phi}(x) = \bar{V}\left(\frac{x}{s_m}\right)^{\frac{4}{3}}$$

$$\bar{E}(x) = -\frac{4}{3}\frac{\bar{V}}{s_m}\left(\frac{x}{s_m}\right)^{\frac{1}{3}}$$

- The Child-Langmuir law

The « real » RF sheath



$$n_e = \begin{cases} 0 & \text{if } s < x \leq s_m; \\ n_i & \text{otherwise,} \end{cases}$$

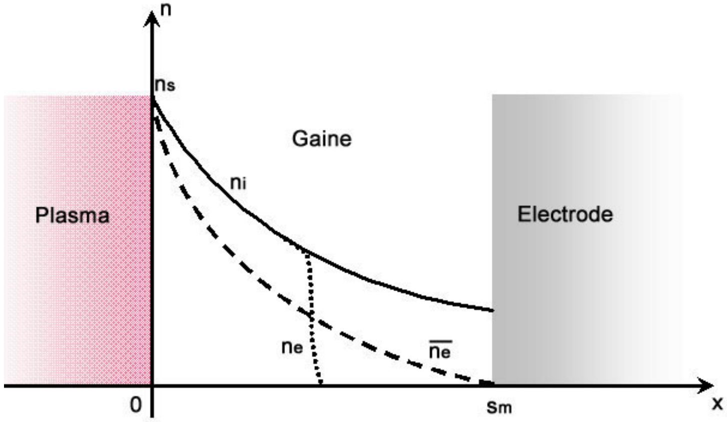
- The ion density profile is independent of time
- It depends of the time-averaged electron density profile because the time-averaged electric field depends on $n_e(t)$
- The self-consistent calculation is difficult
- Lieberman could solve it analytically for sinusoidal waveforms (mathematical complexity)

RF sheaths models

Single frequency, sine wave:

• Valery Godyak, “Soviet Radiofrequency Discharge Research”, Delphic Associates, Fall Church, 1986

• Michael Lieberman, IEEE Plasma Sci. **16** (1988) 638



Dual frequency, sine wave:

• Jérôme Robiche, P C Boyle, M M Turner and A R Ellingboe, J. Phys. D. **36** (2003) 1810

More recently (including kiinetic effects and arbitrary waveforms):

• Brian G Heil et al J. Phys. D: Appl. Phys. **41** (2008) 225208

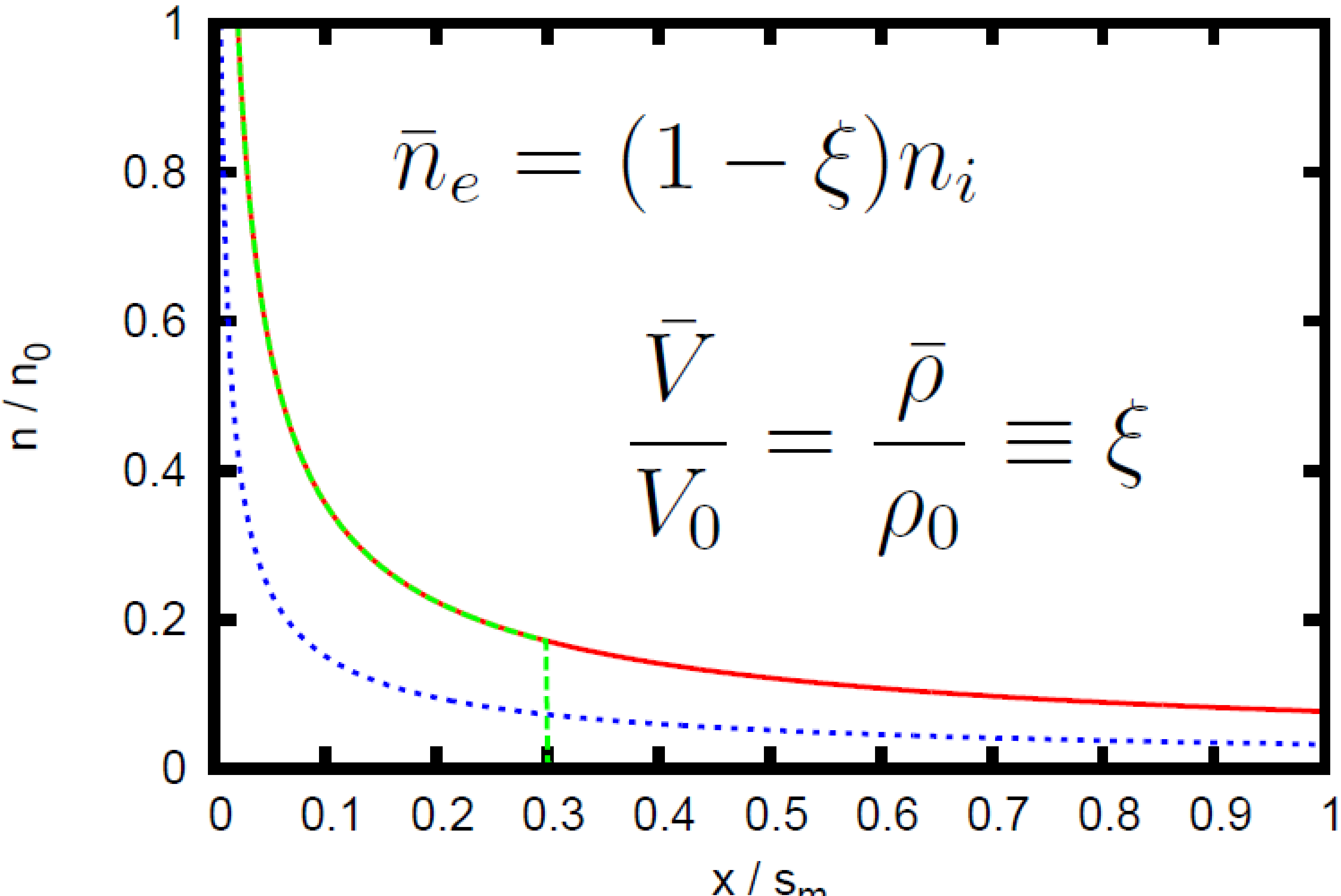
• Mohammed Shihab et al J. Phys. D: Appl. Phys. **45** (2012) 185202

• Uwe Czarnetzki , Physical Review E **88,** 063101 (2013)

• Miles Turner and Pascal Chabert, Appl. Phys. Lett. **104**, 164102 (2014)

• P Chabert and M M Turner, J. Phys. D : Appl. Phys. **50** (2017) 23LT02

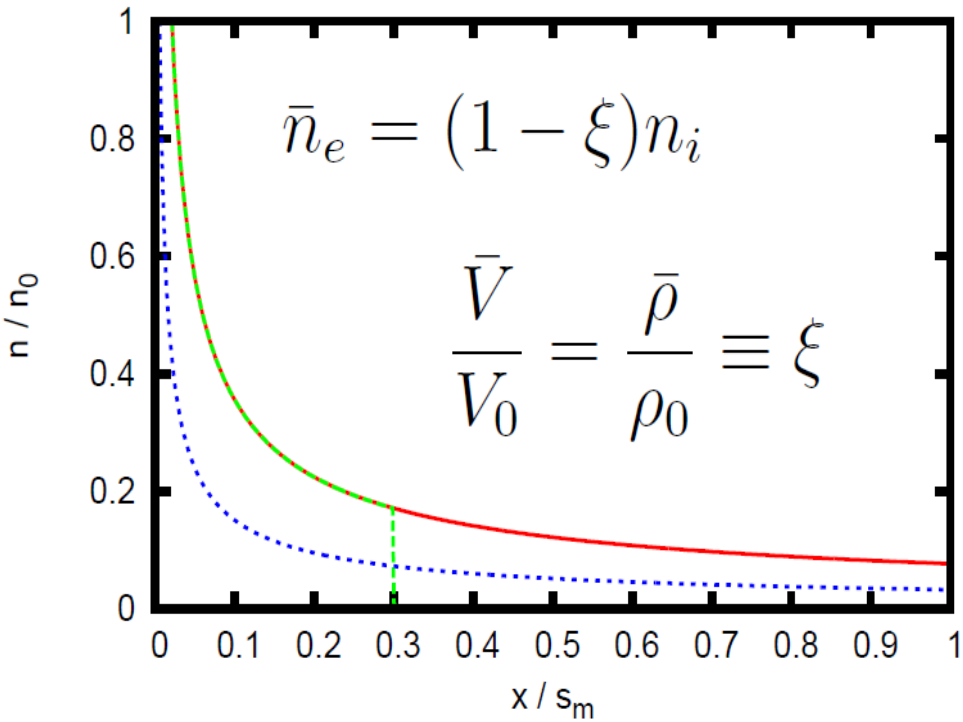
The trick



$$\bar{n}_e = (1-\xi) n_i$$

$$\frac{\bar{V}}{V_0} = \frac{\bar{\rho}}{\rho_0} \equiv \xi$$

Time-averaged quantities



$$J_i = K_i \frac{\epsilon_0}{s_m^2}\left(\frac{2e}{M}\right)^{\frac{1}{2}}\left(-\bar{V}\right)^{\frac{3}{2}}$$

$$n_i(x) = -\frac{4}{9}\frac{\epsilon_0 \bar{V}}{\xi e s_m^2}\left(\frac{s_m}{x}\right)^{\frac{2}{3}}$$

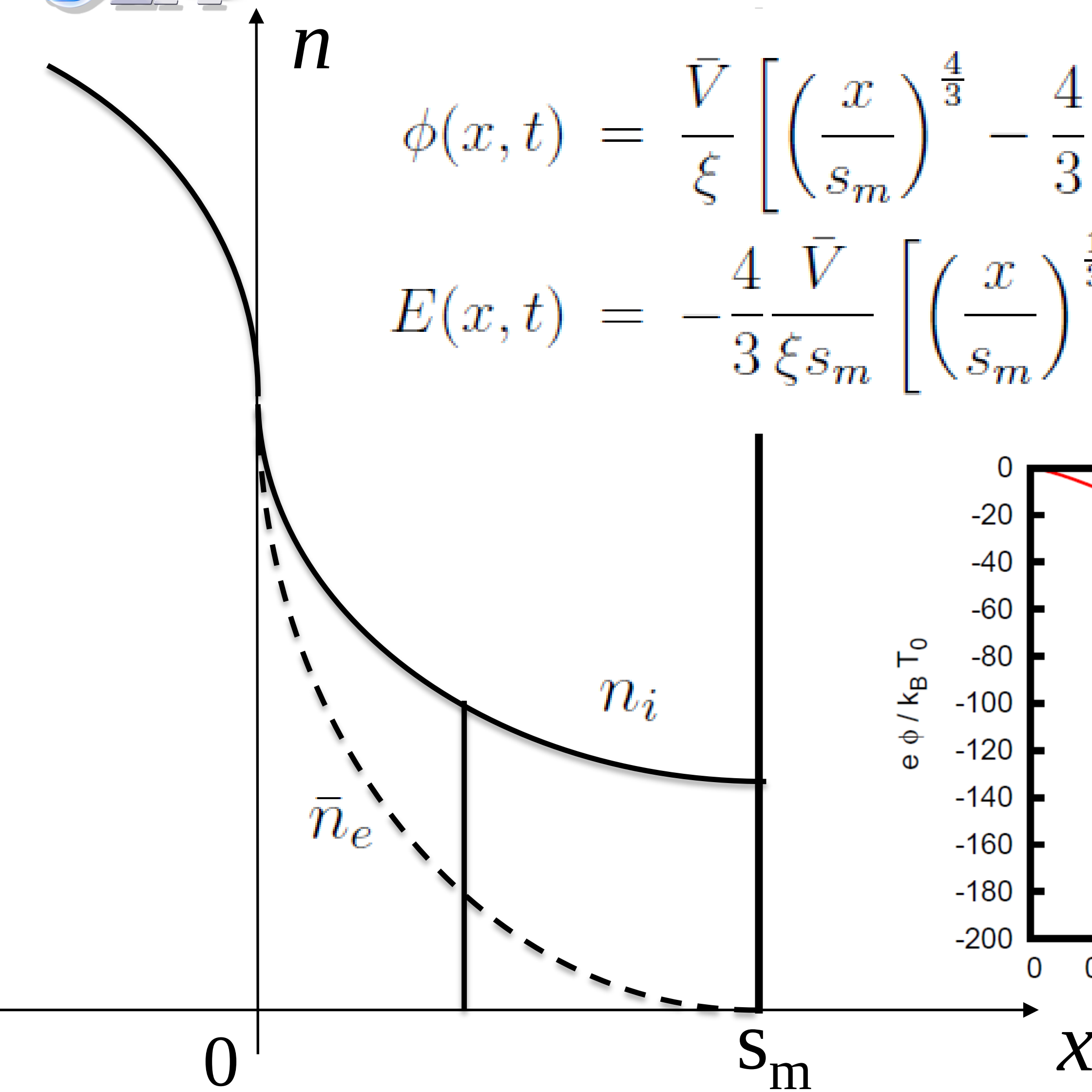
$$\bar{\phi}(x) = \bar{V}\left(\frac{x}{s_m}\right)^{\frac{4}{3}}$$

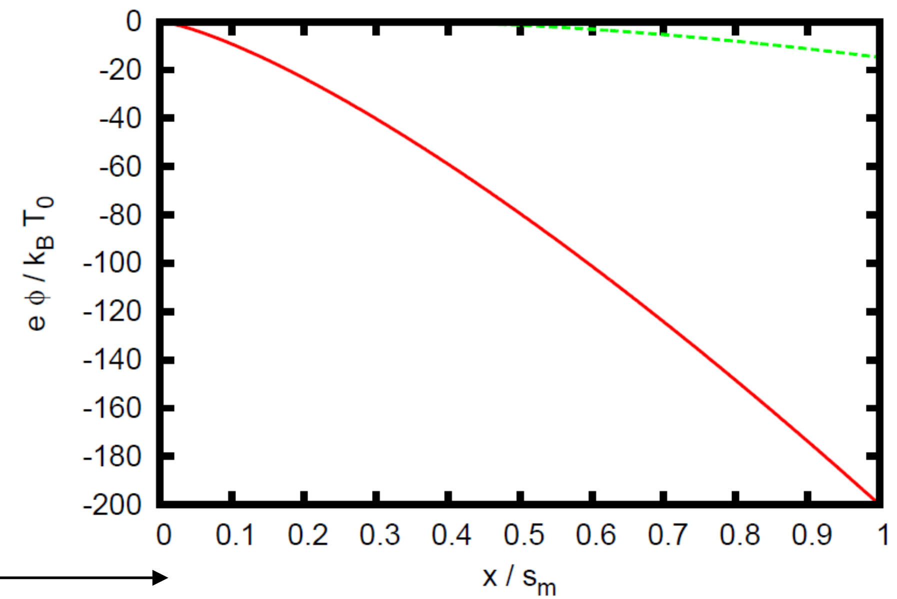
$$\bar{E}(x) = -\frac{4}{3}\frac{\bar{V}}{s_m}\left(\frac{x}{s_m}\right)^{\frac{1}{3}}$$

Time-dependent quantities

$$\phi(x,t) = \frac{\bar{V}}{\xi}\left[\left(\frac{x}{s_m}\right)^{\frac{4}{3}} - \frac{4}{3}\left(\frac{s}{s_m}\right)^{\frac{1}{3}}\left(\frac{x}{s_m}\right) + \frac{1}{3}\left(\frac{s}{s_m}\right)^{\frac{4}{3}}\right]$$

$$E(x,t) = -\frac{4}{3}\frac{\bar{V}}{\xi s_m}\left[\left(\frac{x}{s_m}\right)^{\frac{1}{3}} - \left(\frac{s}{s_m}\right)^{\frac{1}{3}}\right],$$





Find ξ

$$\frac{\bar{V}}{V_0} = \frac{\bar{\rho}}{\rho_0} \equiv \xi \qquad V(t) = V_0\left[1 - \frac{4}{3}\left(\frac{s}{s_m}\right)^{\frac{1}{3}} + \frac{1}{3}\left(\frac{s}{s_m}\right)^{\frac{4}{3}}\right]$$

$$\xi = \frac{\langle V(t)\rangle}{V_0} = \left\langle 1 - \frac{4}{3}\left(\frac{s}{s_m}\right)^{\frac{1}{3}} + \frac{1}{3}\left(\frac{s}{s_m}\right)^{\frac{4}{3}}\right\rangle$$

The procedure

LPP

- Current waveform as the input parameter

- Find s(t): $J = \epsilon_0 \left.\frac{\partial E}{\partial t}\right|_{x=s_m} = \frac{4}{3}\frac{\epsilon_0 V_0}{s_m}\frac{d}{dt}\left(\frac{s}{s_m}\right)^{\frac{1}{3}}$ $\frac{s}{s_m} = \left[\frac{3}{4}\frac{s_m}{\epsilon_0 V_0}\int_0^t J dt\right]^3$

- Find ξ and V(t): $\xi = \frac{\langle V(t)\rangle}{V_0} = \left\langle 1 - \frac{4}{3}\left(\frac{s}{s_m}\right)^{\frac{1}{3}} + \frac{1}{3}\left(\frac{s}{s_m}\right)^{\frac{4}{3}}\right\rangle$

$$V(t) = V_0\left[1 - \frac{4}{3}\left(\frac{s}{s_m}\right)^{\frac{1}{3}} + \frac{1}{3}\left(\frac{s}{s_m}\right)^{\frac{4}{3}}\right]$$

- All other parameters are then easily calculated

Example 1: sine wave

For the single frequency case treated by Lieberman [9] we choose $J(t) = -J_0 \sin \omega t$. From Eqs. (15) and (14) we find

$$s(t) = \frac{s_m}{8}(1 - \cos\omega t)^3 \tag{16}$$

$$J_0 = -\frac{K_{\rm cap}}{2}\frac{\omega\epsilon_0}{s_m}V_0 \tag{17}$$

$$\xi = \frac{163}{384} \tag{18}$$

where $K_{\rm cap} = 4/3$. The voltage waveform is obtained by inserting Eq. (16) into Eq. (12). Combining Eqs. (5) and (17) gives the sheath maximum expansion as a function of J_0,

$$s_m = \frac{K_s J_0^3}{e\epsilon_0 k_B T_0 \omega^3 n_0^2}, \tag{19}$$

where $K_s = 4\xi/3$. These expressions are identical with those of Lieberman [1, 9], apart

Example 1: sine wave

	Present	Lieberman
ξ	0.425	0.415
K_{cap}	1.33	1.23
K_i	1.05	0.82
K_s	0.566	0.417

Example 2: pulsed waveform

As a third example we consider a sheath excited by the pulsed waveform

$$J(t) = J_0 \left(\frac{t}{t_w}\right) \exp\left(\frac{1}{2} - \frac{1}{2}\frac{t^2}{t_w^2}\right), \tag{26}$$

which is representative of several topical experiments [10–12]. We assume that this pulse is repeated at intervals $t_p \ll t_w$, such that successive pulses do not appreciably overlap. In this case we find

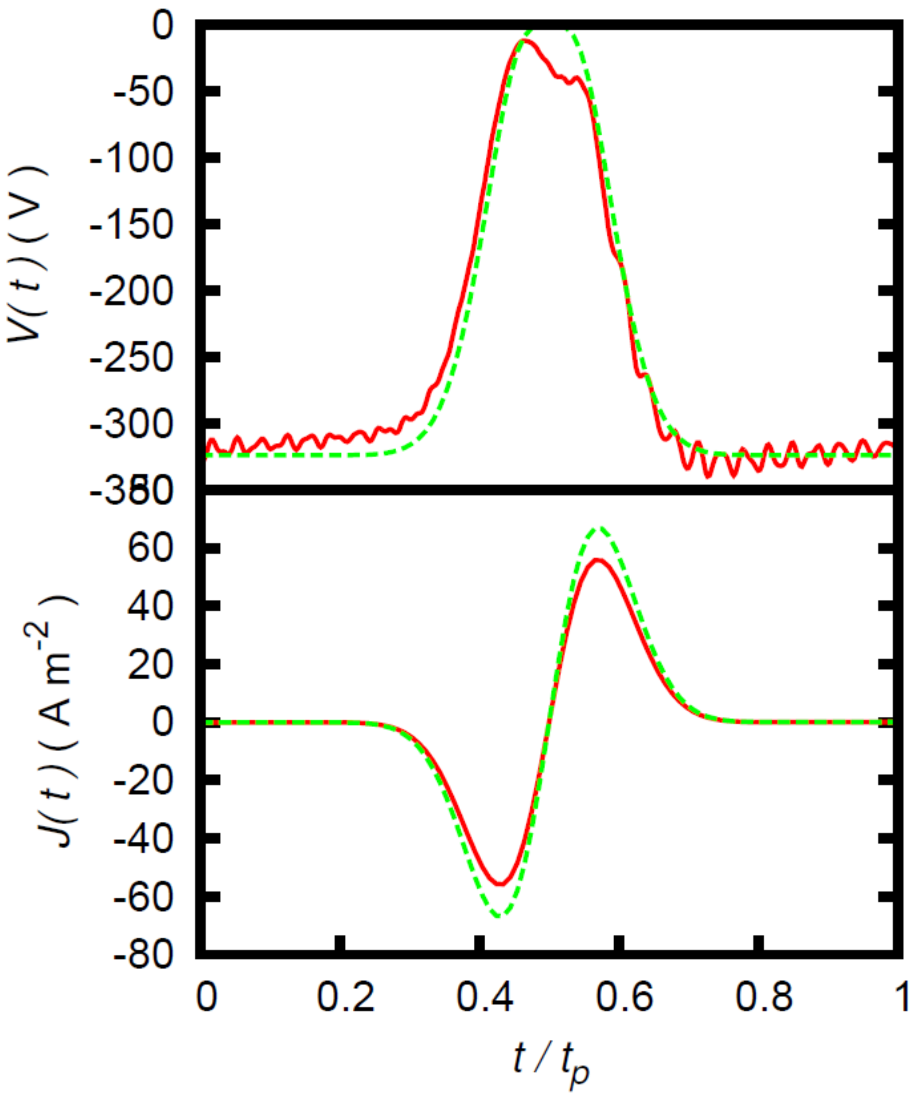
$$s(t) = s_m \exp\left(-\frac{3}{2}\frac{t^2}{t_w^2}\right) \tag{27}$$

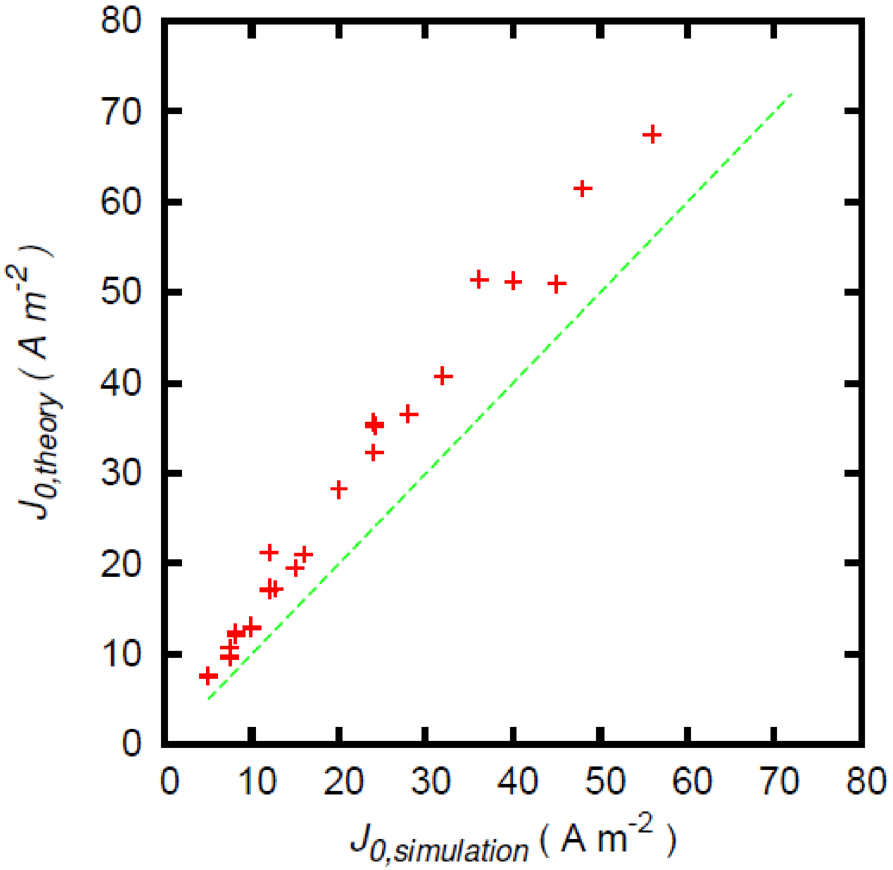
$$\xi = 1 - \frac{7}{3}\sqrt{\frac{\pi}{2}}\frac{t_w}{t_p} \tag{28}$$

$$J_0 = -\frac{4}{3}\frac{\epsilon_0 V_0}{s_m t_w \exp\left(\frac{1}{2}\right)} \tag{29}$$

$$s_m = \frac{\xi}{6}\exp\left(\frac{3}{2}\right)\frac{(J_0 t_w)^3}{\epsilon_0 e n_0^2 k_B T_0} \tag{30}$$

Example 2: pulsed waveform



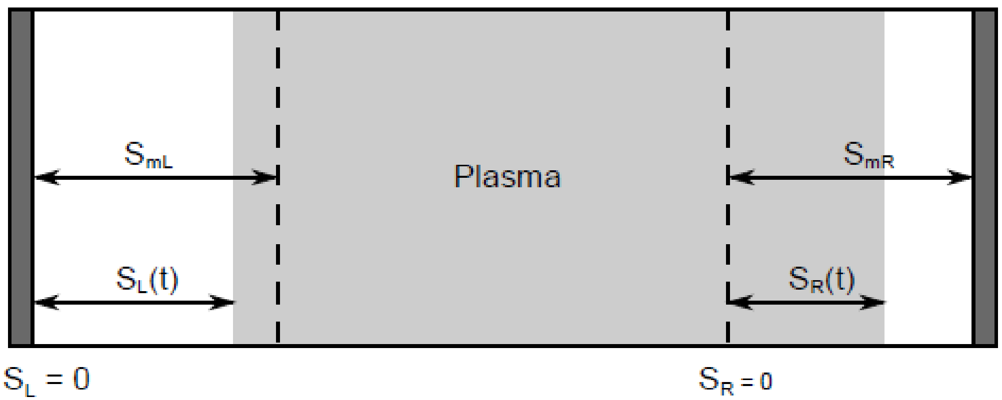


Conclusions

- A sheath model has been developed for arbitrary RF waveforms
- Agrees well with PIC simulations
- Can be extended to two sheaths and then calculate bias formation (see Electrical Assymetry Effect Bochum)
- Heating in the sheath (both collisionless and ohmic) can be calculated analytically: therefore a global model can easily be constructed

The full CCP with arbitrary waveform

Theory for the self-bias formation in capacitively coupled plasmas excited by arbitrary waveforms
T Lafleur, P Chabert, M M Turner and J P Booth
Plasma Sources Sci. Technol. **22** 065013 (2013)



$$\frac{s_R}{s_{mR}} = \alpha_R \left[\frac{\int_{t_0}^{t} I dt}{\int_{t_0}^{t_m} I dt} \right]^3$$

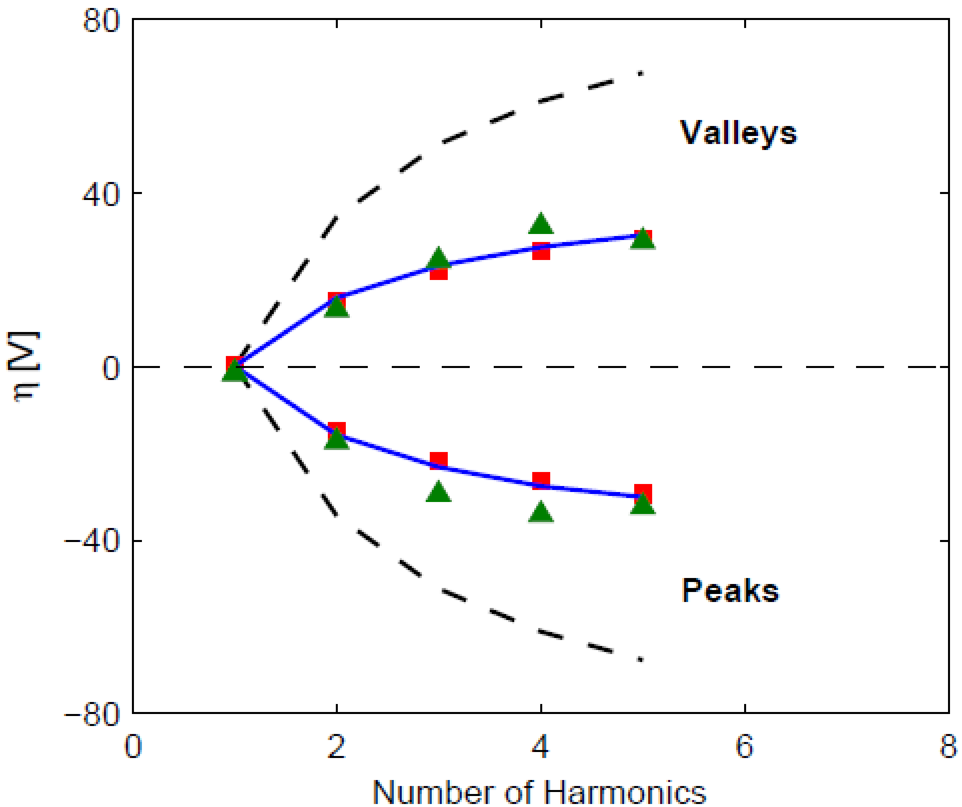
$$\frac{s_L}{s_{mL}} = 1 - (1 - \alpha_L) \left[1 - \frac{\int_{t_0}^{t} I dt}{\int_{t_0}^{t_m} I dt} \right]^3$$

$$\frac{V_R}{V_{0R}} = \left[1 - \frac{4}{3} \left(\frac{s_R}{s_{mR}} \right)^{1/3} + \frac{1}{3} \left(\frac{s_R}{s_{mR}} \right)^{4/3} \right]$$

$$\frac{V_L}{V_{0L}} = \left[1 - \frac{4}{3} \left(\frac{s_{mL} - s_L}{s_{mL}} \right)^{1/3} + \frac{1}{3} \left(\frac{s_{mL} - s_L}{s_{mL}} \right)^{4/3} \right]$$

Self-bias

$$\eta = \frac{1}{T}\int_0^T V_{rf}\,dt = \frac{1}{T}\int_0^T (V_L - V_R)\,dt = \xi_L V_{0L} - \xi_R V_{0R}$$



- Dashed line is when conduction current balance is not satisfied
- Excellent agreement with PIC results if conduction current balance is satisfied (blue line)

$$n_0 u_B = \frac{1}{T}\int_0^T \frac{1}{4} n_0 \bar{v}_e e^{V_R/T_e}\,dt$$

$$n_0 u_B = \frac{1}{T}\int_0^T \frac{1}{4} n_0 \bar{v}_e e^{V_L/T_e}\,dt$$

Plasma potential

- Plasma potential waveform can be obtained analytically

$$V_p = -V_R$$

$$V_{rf} = V_L + V_p = V_L - V_R$$

- Excellent agreement with PIC results