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Debye sheath: comparison between fluid predictions & kinetic simulations

Towards a gyrokinetic modeling

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Physical context: improved confinement in tokamak plasmas largely governed by edge physics

- Energy confinement in tokamak plasmas governed by
 turbulent transport across magnetic field surfaces
- Most regimes of improved confinement characterized by transport barriers close to last closed field surface [Wagner 1982, Whyte 2010, ...]
- □ Critical role of the **sheared radial electric field** E_r (→ sheared rotation of turb. eddies) in turbulence regulation

[Itoh-Itoh 1988, Biglari-Diamond-Terry 1990, ...]



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[Itoh-Itoh 1988, Biglari-Diamond-Terry 1990, ...]

- Several players involved in **E**, **profile**:
 - Confined region: radial force balance $E_r = \frac{\nabla_r p_{\perp i}}{2} V_{\theta} B_{\varphi} + V_{\varphi} B_{\theta}$
 - *Edge*: ion orbit losses, collisional drag
 - SOL: plasma-wall interaction $E_r^{SOL} \approx -\frac{\Lambda}{2} \nabla_r T_e$
 - All: turbulence (Reynolds stress)





Need for **core-edge-SOL modeling** to understand & predict bifurcations toward improved confinement regimes





- **1. Kinetics of plasma-wall interaction**: analogies with & departures from fluid predictions (VOICE)
- 2. Coupling kinetic plasma species & fluid neutrals proof of principle (VOICE)

3. Towards the implementation of plasma-wall interaction in 5D gyrokinetics (GYSELA)

VOICE = (1D,1V) – fully kinetic electrons & ions – Poisson **GYSELA** = (3D,2V) – GK ions, DK electrons – Quasi-Neutrality

Flux-driven, semi-Lagrangian



Objective

Understanding (1D,1V) VOICE results from a fluid perspective:

Plasma self-organization under the **balance of sources, collisions** and **parallel transport losses** mediated at the plasma-wall Debye sheath



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Focus on numerical issues: semi-Lagrangian + non-equidistant mesh

□ Strang splitting → solve Vlasov advections & Source/Collisions separately [Cheng-Knorr 1976, Strang 1968] $\partial_t f_a + \sqrt{A_a} (v_a \partial_x f_a - Z_a \partial_x \phi \ \partial_{v_a} f_a) = C(f_a) + S(f_a)$

Semi-Lagrangian scheme [Staniforth 1991, Sonnendrücker 1999]

 $Vlasov \Rightarrow f_s = Cst along trajectories$

- Find foot print (x_i^{*},v_i^{*}) of characteristics from (x_i,v_i)
- Interpolation (cubic splines) $\rightarrow f_s(x_i^*, v_i^*, t)$

Refined mesh where strong gradient/curvature [Bourne 2023]

Typical parameters:

 $m_i/m_e = 400$ mass ratio $L_x = 700\lambda_{D_0}$ sim. box length $N_x = 1500$ pts in x direction $N_v = 700$ pts in v direction

~20h to reach steady state (NVIDIA Tesla V100, 5120 CUDA cores)





Focus on collision operator $\partial_t f_a + \sqrt{A_a} (v_a \partial_x f_a - Z_a \partial_x \phi \ \partial_{v_a} f_a) = \mathcal{C}(f_a) + \mathcal{S}(f_a)$

Self collisions

Magnitude

$$\mathcal{C}_{ss}(f_s) = \nu_{\mathrm{D0}}^{\star} \frac{\partial}{\partial v_s} \left\{ D_v f_{Ms} \frac{\partial}{\partial v_s} \left(\frac{f_s}{f_{Ms}} \right) \right\}$$

Velocity dependent

diffusion coef. ~ v^{-3}

[Dif-Pradalier 2011, Estève 2015]

Maxwellian with space-time evolving N_{Ms} , U_{Ms} & T_{Ms} to conserve particles, momentum & energy

$$D_{v} = D_{0} \frac{\Phi - G}{y_{s}} \qquad \left| \begin{array}{cc} \Phi(y_{s}) &=& \frac{2}{\sqrt{\pi}} \int_{0}^{y_{s}} e^{-z^{2}} dz \\ G(y_{s}) &=& \frac{\Phi - y_{s} \Phi'}{2y_{s}^{2}}. \end{array} \right| \qquad y_{s} = |v_{s}|/\sqrt{2T_{s}}$$

$$\Box \text{ Inter species coll. } \mathcal{C}_{ss'}(f_s) = \nu_{D0}^{\star} \left\{ \frac{C_{\mathcal{E}}^{ss'}}{n_s T_s} \left(\frac{(v_s - u_s)^2}{T_s} - 1 \right) + \frac{C_{\Gamma}^{ss'}}{n_s T_s^{1/2}} \frac{v_s - u_s}{\sqrt{T_s}} \right\} F_{Ms} \\ C_{\mathcal{E}}^{ss'} = -3n_s \frac{m_s}{m_s + m_{s'}} \nu_{ss'} (T_s - T_{s'}) - u_s C_{\Gamma}^{ss'} \qquad C_{\Gamma}^{ss'} = -n_s m_s \nu_{ss'} (u_s - u_{s'})$$

 \Rightarrow Simple fluid-like momentum & energy transfers

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Focus on source/sink terms $\partial_t f_a + \sqrt{A_a} (v_a \partial_x f_a - Z_a \partial_x \phi \ \partial_{v_a} f_a) = \mathcal{C}(f_a) + \mathcal{S}(f_a)$

normalization to $\lambda_{D0} = (\varepsilon_0 T_0 / e^2 n_0)^{1/2}$, $v_{T0s} = (T_0 / m_s)^{1/2}$, $\omega_{p0s} = v_{T0s} / \lambda_{D0}$



N.B.: \mathcal{M}_{w} does not precisely determine wall position x_{w}

boundary condition

Collisions mandatory to reach steady-state

Electron energy conservation at vanishing source & without collisions:

$$K = \frac{1}{2}v^2 = K_0 - (\phi_0 - \phi)$$





Atelier Gaine Plasma – Marseille – 4-6/11/2024

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Distribution functions @ sheath entrance ≠ truncated Maxwellians

Plasma self-organization under the combined effects of source, collisions and Debye sheath



Electrons: collisions + incomplete wall absorption

Recovery of potential drop & its expected dependencies

Detential drop in the sheath $\Delta \phi_{sh}$ governed by different electron-ion inertia

$$\Delta \phi_{\rm sh}^{\rm pred.} = \frac{T_{\rm e}^{\rm sh}}{e} \log \left(\frac{\Gamma_{\rm i}^{\rm sh}}{\Gamma_{\rm e}^{\rm sh+}} \right) = \frac{T_{\rm e}^{\rm sh}}{2e} \log \left(2\pi \frac{m_{\rm e}}{m_{\rm i}} M^2 \left(1 + \frac{T_{\rm i}^{\rm sh}}{T_{\rm e}^{\rm sh}} \right) \right)$$
$$2\Lambda \approx 2 \times 3.8$$

□ Recovered with VOICE provided one accounts for "outgoing" electrons $\Gamma_e^ \Gamma_e^{\text{sh}}$ $\Gamma_e^{+\infty}$ $\Gamma_e^{-\frac{m_e v^2}{2T_e^{\text{sh}}}}$

$$\Rightarrow \Gamma_{\rm i}^{\rm sh} = \Gamma_{\rm e}^{+\rm pred.} - \Gamma_{\rm e}^{-} \qquad \text{With } \Gamma_{\rm e}^{+\rm pred.} = \int_{v_{\rm c}}^{+\infty} dv \, v n_e^{\rm sh} \sqrt{\frac{m_{\rm e}}{2\pi T_{\rm e}^{\rm sh}}} e^{-2T_{\rm e}^{\rm sh}}$$

So that

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$$\Delta \phi_{\rm sh}^{\rm pred, \, ref.} = \frac{T_{\rm e}^{\rm sh}}{e} \log \left(\sqrt{2\pi \frac{m_{\rm e}}{m_{\rm i}} M^2 \left(1 + \frac{T_{\rm i}^{\rm sh}}{T_{\rm e}^{\rm sh}}\right)} \left(1 + \frac{\Gamma_{\rm e}^-}{\Gamma_{\rm i}^{\rm sh}}\right) \right)$$

□ Important consequence for fusion plasmas:

Radial electric field positive in the Scrape-Off Layer (SOL) $E_r^{SOL} \approx -\frac{\Lambda}{2} \nabla_r T_e > 0$





Bohm criterion: depends on – not well defined – sound speed

□ Fluid prediction for plasma-wall interaction:

Bohm criterion: $M = u_i/c_s \ge 1$ at sheath entrance \Rightarrow supersonic ion flow in Debye sheath

• Expression of c_s depends on fluid closure



From Fourier transform of fluid equations \rightarrow dispersion relation for c_s

Closure	Isothermal	Maxwellian	Cold ions	Polytropic
Sound speed <i>c</i> _s	$\sqrt{\frac{T_i + T_e}{m_i}}$	$\sqrt{\frac{3(T_i + T_e)}{m_i}}$	$\sqrt{rac{T_e}{m_i}}$	$\sqrt{\frac{\gamma_p(T_i+T_e)}{m_i}}$
Assumptions	$T_s = cte.$	$Q_s^{\text{heat}} = 0$	$T_i = 0$	$\frac{dp}{p} = \gamma_p \frac{dn}{n}$
$Q_s^{heat} = \int_{-\infty}^{+\infty} dv rac{1}{2} m_s (v-u_s)^3 f_s$				

□ VOICE results: $Q_s^{\text{heat}} \neq 0$

Bohm criterion $M = \pm 1$ is not operational to define

Debye sheath entrance

Larger sheath heat transmission factors than predicted

Total energy flux expressed as a function of convected flux:

$$Q_s = \gamma_s \Gamma_{\rm sh} T_s$$

Heat transmission factor

 $\hfill\square$ Fluid framework (Maxw. closure) \rightarrow Prediction for γ_s

VOICE results: larger γ_i (~60%) and γ_e (~35%)





Ginetic results:

• Non vanishing heat flux (not predicted in fluid)

$$Q_{s} = \underbrace{u_{s}\left(\frac{3}{2}n_{s}T_{s} + \frac{1}{2}m_{s}n_{s}u_{s}^{2}\right)}_{\mathbf{Q}_{\text{conv,e}}} + \underbrace{Q_{s}^{heat}}_{\mathbf{Q}_{s}}$$

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• Extremely slow convergence of Q_s^{heat} towards 0 when $v_0^* \uparrow$



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Accounting for a source of neutrals

Motivations

• Plasma-wall interaction: particles recycled by the wall as neutrals at a rate > 99% in W environment

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- SOL turbulent dynamics mostly governed by convection
- Flux-driven GK simulations with kinetic electrons require a particle source

- Physics of the plasma-neutral interaction
 - Considered reactions: charge-exchange, ionization & recombination

 $\langle \sigma v \rangle_i$ strongly increases from 1 to 10eV \Rightarrow threshold $\langle \sigma v \rangle_r$ only relevant below T \approx 1eV

- Particle source/sink $\rightarrow \langle \sigma v \rangle_i \& \langle \sigma v \rangle_r$
- Momentum & Energy transfers $\rightarrow \langle \sigma v \rangle_{CX}, \langle \sigma v \rangle_{i} \& \langle \sigma v \rangle_{r}$





Neutrals modelled as a fluid \rightarrow coupling to kinetic plasma species

- Neutral-plasma coupling via source terms
 - So far, source/sink restricted to particles (ioniz. + recomb.)
 - Constraint: ensure proper balance "1 neutral \leftrightarrow 1 ion + 1 electron"

General method: projection on Hermite (& Laguerre) polynomials

[Sarazin 2011]

•
$$S_{v}(v_{a}) = \sum_{h=0}^{+\infty} c_{h} H_{h}\left(\frac{v_{a}}{\sqrt{2T_{sc}}}\right) e^{-\frac{v_{a}^{2}}{2T_{sc}}} \rightarrow \text{Fluid moments related to } c_{h} \text{ coefficients}$$

• Pure Sce of particles: $(S_{n}(v_{a}) = c_{0}\left(\frac{3}{2} - \frac{v_{a}^{2}}{2T_{sc}}\right) e^{-\frac{v_{a}^{2}}{2T_{sc}}} \text{ with } \begin{bmatrix} c_{0} = \frac{S_{n,N}}{\sqrt{2\pi T_{sc}}} \\ S_{n,N} = n_{N}n_{e}\langle\sigma v\rangle_{i} - n_{i}n_{e}\langle\sigma v\rangle_{r} \\ \int_{-\infty}^{+\infty} dv_{a} S_{n}(v_{a}) = \sqrt{2T_{sc}} \sum_{h} \langle H_{0}, c_{h}H_{h} \rangle = \sqrt{2\pi T_{sc}} c_{0} = S_{n,N} \qquad \langle f,g \rangle = \int_{-\infty}^{+\infty} f(y)g(y)e^{-y^{2}}dy \end{bmatrix}$
Conservation equations: $\begin{bmatrix} \frac{df_{a}}{dt} = C(f_{a}) + S_{n}(v_{a}) \\ \partial_{t}n_{N} + \nabla \cdot \Gamma_{N} = -S_{n,N} \end{bmatrix}$

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Plasma-wall interaction in the GK framework: main issues

Debye sheath physics

- Non-neutral plasma
- Strong parallel electric field on a few $\lambda_{\rm D}~\rightarrow$
- Main objectives / challenges
 - Keep the plasma quasi-neutral (k λ_D <<1)
 - Recover linear dependency of ϕ with T_e
 - Ensure absorption of all ions & reflection of slow electrons

State-of-the-art

- Logical sheath
- Conducting sheath
- New model: "flux-averaged sheath"

adapted to semi-Lagrangian full-f

 $\begin{array}{l} & \mbox{GK physics} \\ & \mbox{violates usual GK assumption} \\ & \mbox{subgrid w.r.t. typical scales } L_{//} \approx L_s/k_{\theta}\rho_s \end{array}$ OK with adiabatic electrons $\begin{array}{l} & \mbox{[Caschera 2018, Dif-Pradalier 2022]} \end{array}$



Constraining the cut-off velocity $v_c = \sqrt{(2e \Delta \phi/m_e)}$

Electrons with $|v_{//}| \le v_c$ are reflected back \rightarrow different strategies to estimate $v_c(\Delta \phi)$

• Logical sheath (initially developed for PIC codes): [Parker 1993] enforces j=0 at each time/position on $\partial \Omega$

- At each time, count the N_{i} ions crossing $\partial \Omega$
- Remove the N_i fastest electrons \rightarrow v_c is the velocity of the fastest reflected electron

• Conducting sheath: allows for finite local currents on $\partial \Omega$ [Shi 2015]

- GK quasi-neutrality $\nabla_{\perp}^2 \phi = \rho \rightarrow \phi$ at any position on $\partial \Omega$
- $v_c = \sqrt{(2e \Delta \phi/m_e)}$ defines the velocity of the slowest absorbed electron





(I)

Constraining the cut-off velocity $v_c = \sqrt{(2e \Delta \phi/m_e)}$

Electrons with $|v_{//}| \le v_c$ are reflected back \rightarrow different strategies to estimate $v_c(\Delta \phi)$

"Flux-averaged sheath": [Munschy 2024 c]



(11)

• Enforces *vanishing current* **on** *average over* $\partial \Omega \rightarrow$ allows for **finite local currents**

•
$$\mathbf{v}_{c}$$
 implicitly defined by
$$\left\langle \Gamma_{i}^{\partial\Omega}(\mathbf{x},t)\right\rangle_{\partial\Omega} = \left\langle \bar{\Gamma}_{e}^{\partial\Omega}(\mathbf{x},v_{c},t)\right\rangle_{\partial\Omega} \approx \mathbf{B}/B \approx \mathbf{$$

- All crossing ions are absorbed (penalization)
- Slow electrons are reflected back into the plasma

N.B.: additional complexity arising from backward semi-Lagrangian scheme



Plasma-wall boundary: ions absorbed & electrons reflected

Simulations w/o quasi-neutrality \rightarrow successful check of ion & electron dynamics

Ions absorbed within the limiter via penalization \rightarrow mask M_{lim} $dF_i/dt = ... - v (F_i - F_{lim}) M_{lim}$

Electrons reflected back at the cut-off velocity v_c

Depletion front propagates along θ away from limiter / into the plasma



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Electrons reflected back at the cut-off velocity v_c

Depletion front propagates along θ away from limiter / into the plasma

Current issue with QN:

Charge density build-up close to limiter due to imbalance between ion & electron particle fluxes



Thin toroidal limiter = "simple" alternative Boundary Condition

Axisymmetric limiter



Particles intercept **axisymmetric** limiter during (r, θ) advection

⇒ immersed in Vlasov and Quasi-Neutrality

- Strang splitting \rightarrow separate advections in (r, θ) and ϕ
- *1.* v_{\parallel} advection on $\Delta t/2$
- *2.* φ advection on $\Delta t/2$
- *3.* (r, θ) advection on Δt
- *4.* φ advection on $\Delta t/2$
- 5. v_{\parallel} advection on $\Delta t/2$

Thin limiter located in between

two toroidal mesh points



Particles intercept **thin** limiter during φ advection

⇒ immersed in Vlasov only (not in Quasi-Neutrality)

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Thin toroidal limiter \rightarrow depletion of SOL density at early times



Conclusions



- Kinetic Debye sheath (VOICE)
 - Immersed boundaries (penalization) OK to recover main physics
 - Collisions mandatory \rightarrow allow for steady-state non "ideal" distribution functions
 - C_s loosely defined \Rightarrow Bohm criterion not operational (\rightarrow Debye sheath where ρ shoots-up)
 - Kinetic self-organization \rightarrow finite ion & electron heat fluxes \Rightarrow large sheath heat transmission factors

• Neutrals as a fluid (VOICE \rightarrow Gysela-X)

- Fluid model successfully coupled to kinetic plasma
- Next steps: Add momentum & energy exchanges Account for recycling coefficients (> 99% for particles in W environment) $\rightarrow S_{n,N} \propto \Gamma_i^{\partial\Omega}$ From 1D to 3D \rightarrow implement in Gysela-X

Consistent plasma-wall interplay within Gyro-Kinetic description (GYSELA)

- Subgrid modelling of Debye sheath is challenging
- Absorption of ions & reflection of slow electrons OK in GYSELA
- New "Flux-averaged sheath" model current issue (flux imbalance) under investigation
- Promising alternative: toroidally localized limiter \rightarrow requires adjustments

Associated publications



VOICE results

E. Bourne et al., *Non-uniform splines for semi-Lagrangian kinetic simulations of the plasma sheath* J. Comput. Phys. 488 (2023) 112229

Y. Munschy (a) et al., *Kinetic plasma-wall interaction using immersed boundary conditions* Nucl. Fusion 64 (2024) 056027

Y. Munschy (b) et al., *Kinetic plasma-sheath self-organization* Nucl. Fusion 64 (2024) 046013

GYSELA results

E. Caschera et al., *Immersed boundary conditions in global, flux-driven, gyrokinetic simulations* J. Phys. Conf. Series 1125 (2018) 012006

G. Dif-Pradalier et al., *Transport barrier onset and edge turbulence shortfall in fusion plasmas* Commun. Phys. 5 (2022) 259

Y. Munschy (c), *Kinetic and Gyrokinetic physics of plasma-wall interaction in tokamaks* PhD thesis, Aix-Marseille Univ. (2024)